# CIVIL SERVICES EXAMINATION (MAINS) 2019

# DIAS MATHEMATICS OPTIONAL TEST SERIES

# TEST 2

**Time Allowed: 3 Hours** **Max Marks: 250**

**Instructions**

- **There are 8 questions divided in 2 sections**
- **Attempt 5 questions in all**
- **Questions No 1 and 5 are compulsory and out of the remaining any 3 are to be attempted choosing atleast 1 question from each section.**

## Section – A

Q1 (a) If H is a subgroup of a group G such that $x^2 \in H$ for every x $\in G$ then prove that H is a normal sub group of G (10 marks)

(b) Let $x_1$ = 2 and $x_{n+1}$= $\sqrt{x_n + 20}$ , n = 1, 2, 3 show that sequence $x_1, x_2, x_3$ is convergent. (10 marks)

(c) $\oint_c \frac{e^{2z}}{(z+1)^4}$ dz where c is circle $|z|$ = 3 (10 marks)

(d) If $f(z) = u + iv$ is analytic function of complex variable z and u- v = $e^x$ $(\cos y - \sin y)$ determine $f(z)$ in terms of z (10 marks)

(e) Given the programme

Maximize u = 5x + 2y

Subject to x + 3y ≤ 12

3x – 4y ≤ 9

7x + 8y ≤ 20

x, y ≤ 0

Write its dual in standard form (10 marks)

Q2 (a) (i) Show that any two cyclic group of same order are isomorphic. (10 marks)

(ii) Show that group of order 15 is cyclic (10 marks)

(b) Define $\{x_n\}$ by $x_1$ = 5 and $x_{n+1}$ = $\sqrt{4 + x_n}$ , for n > 1 show that sequence converges to $\frac{1+\sqrt{7}}{2}$ (15 marks)

(c) Using contour integration to prove

$\int_0^{\pi} \frac{a\, d\theta}{a^2+Sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}$ , a > 0 (15 marks)

Q3 (a) Show that set $\{a + b\omega : \omega^3 = 14$ where a and b are real number in a field with respect to usual addition & multiplication
(15 marks)

(b) Is the function

$$f(x) = \begin{cases} \frac{1}{n}, \frac{1}{n+1} < x \leq \frac{1}{n} \\ 0, x = 0 \end{cases}$$

Remain integrable? If yes, obtain value of $\int_0^1 f(x)dx$ (15 marks)

(c) Evaluate $\int_0^{\pi} \frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$ using residue (20 marks)

Q4 (a) Use simplex method to solve LPP

Max Z = $4x_1$ + $10x_2$

Subject to $2x_1 + x_2 \leq 50$

$2x_1$ + $5x_2 \leq 100$

$2x_1$ + $3x_2 \leq 90$

$x_1, x_2 \geq 0$ (15 marks)

(b) Find optimal assignment for assignment cost

| | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 15 | 7 | 9 |
| 2 | 14 | 10 | 12 |
| 3 | 15 | 13 | 16 |

(15 marks)

(c) Find the absolute maximum & minimum values of $f\ (x, y)$ = $e^{xy}$ on domain $x^2$ + $2y^2 \leq 1$. Check for critical point in interior of the domain. (20 marks)

## Section - B

Q5 (a) Form pde by elementary arbitrary function $\varphi$ and $\psi$ from the relation

Z = $\varphi$ ($x^2$ – y) + $\psi$ ($x^2 + y$) (10 marks)

(b) Solve pq = $x^m\ y^n\ z^{2l}$ (10 marks)

(c) Find equation of motion of compound pendulum using Hamilton's equation (10 marks)

(d) Use newton's iterative formula establish the iterative formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right)$

Calculate cube root of N (10 marks)

(e) Show that $\emptyset = (x-t)\ (y-t)$ represent the velocity potential of an incompressible two dimensional fluid further show that the streamlines at the time t are curves.

$(x-t)^2 - (y-t)^2$ = constant (10 marks)

Q6 (a) Reduce $\frac{\partial^2 z}{\partial x^2} = x^2\ \frac{\partial^2 z}{\partial y^2}$ to canonical form (15 marks)

(b) Find value of $\int_1^5 log_{10}\, x\ \ dx$ taking 8 sub interval correct to 4 decimal places by trapezoidal rule (15 marks)

(c) Two sources each of strength m are placed at point (-a, 0) , (a, 0) and a sink of strength 2m is at origin. Show that the stream lines are the curve.

$(x^2 + y^2)^2 = a^2\ (x^2 - y^2 + \lambda\, xy)$ where $\lambda$ is variable parameter. Show also that fluid speed at any point is $\frac{2\, m\, a^2}{r_1\, r_2\, r_3}$ where $r_1\ r_2\ r_3$ are distance of point from source and sink. (20 marks)

Q7 a Convert decimal number to equivalent binary to hexadecimal number

(i) 4096
(ii) 0.4375
(iii) 2048.0625 (15 marks)

b Solve $\frac{\partial^3 u}{\partial n^3} + \frac{\partial^3 u}{\partial y^3} + \frac{\partial^3 u}{\partial z^3} - \frac{3\partial^3 u}{\partial x \partial y \partial z} = x^3 + y^3 + z^3 - 3xyz$

c. Using Runga Kutta method find an approximate value of y from x = 0.2 if $\frac{dy}{dx} = x + y^2$ gives that y= 1 when x = 0 (20 marks)

Q8 a) Find the characteristic of equation of pq = z and determine integral surface which passes through parabola x = 0, $y^2$ = z (20 marks)

b) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ under the condition

(i) u(0, t) = u(2, t) = 0, t>0

(ii) u(x, 0) = $Sin^3\left(\frac{\pi x}{2}\right)$, $0 \leq x \leq 2$

(iii) $\left.\frac{\partial u}{\partial t}\right|_{t=0}$ = 0 (15 marks)

C A uniform rod OA of length 2a free to turn about its end O revolve with uniform angular velocity $\omega$ about vertical OZ through O and is inclined at constant angle $\alpha$ to OZ .Show that value of $\alpha$ is either zero or $Cos^{-1}\left(\frac{3g}{4aw^2}\right)$ (15 marks)