CIVIL SERVICES EXAMINATION (MAINS) 2019

DIAS MATHEMATICS OPTIONAL TEST SERIES

TEST 1

Time Allowed: 3 Hours **Max Marks: 250**

Instructions

- **There are 8 questions divided in 2 sections**
- **Attempt 5 questions in all**
- **Question No 1 and 5 are compulsory and out of the remaining, any 3 are to be attempted choosing atleast 1 question from each section.**

Section A

Q1 (a) Find the inverse of matrix

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$$

By using elementary row operation. Hence solve the system of linear equation

x + 3y + z = 10

2x – y + 7z = 21

3x + 2y – z = 4 (10 marks)

(b) Show that mapping T: $R^3 \to R^3$ defined by T (x, y, z) = (x – y + 2z, 2x +y, -x -2y +2z) is a linear transformation. Find its nullity. (10 marks)

(c) Find value of

$$\lim_{n \to \infty} \frac{[(n+1)(n+2)\ldots\ldots(n+n)]^{1/n}}{n}$$ (10 marks)

(d) A plane passes through a fixed point (a, b, c) and cut the axes in A, B, C. show that the locus & centre of sphere OABC where O is origin is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$ (10 marks)

(e) If xyz = 8 find value of x, y, z for which

$u = \frac{5xyz}{x+2y+4z}$ is maximum (10 marks)

Q2 (a) (i) Evaluate limit

$$\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$ (12 marks)

(ii) Prove that $\int_{\partial}^{\infty} \frac{tan^{-1}\alpha x \ tan^{-1}\beta x}{x^2} \ dx = \frac{1}{2}\pi \ log \left\{\frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^{\alpha}\beta^{\beta}}\right\}$ (13 marks)

(b) (i) Find the reciprocal of matrix T = $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ then show that the transform of the matrix A = $\frac{1}{2}\begin{pmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{pmatrix}$ by T i.e. TAT^{-1} is a diagonal matrix. Determine eigen value of matrix A. (8 marks)

(ii) Prove that the equation ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0 represent a cone if $\frac{u^2}{a}$ + $\frac{v^2}{b}$ + $\frac{w^2}{c}$ = d
(7 marks)

(c) Show that lines drawn from the origin parallel to normal to the central conicoid ax^2 + by^2 + cz^2 = 1 at its point of intersection with plane lx + my + nz = p generate the cone

$p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right) = \left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$ (10 marks)

Q3 (a) (i) Show that if A and B are similar $n \times n$ matrices, then they have same eigen value. (8 marks)

(ii) Let u(x, y) = ax^2 + 2 h xy + by^2 and v(x, y) = Ax^2 + 2Hxy + By^2. Find the jacobian J = $\frac{\partial(u,v)}{\partial(x,y)}$ and show that u, v are independent unless

$\frac{a}{A} = \frac{b}{B} = \frac{h}{H}$ (7 marks)

b (i) Find area of surface of sphere $x^2 + y^2 + z^2 = a^2$ which lies inside cylinder $x^2 + y^2$ = ay (p8 marks)

(ii) IF lx + my + nz = 0 is the diametral plane of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$, prove that area of central plane is $\frac{\pi abc\sqrt{l^2+m^2+n^2}}{\sqrt{a^2l^2+b^2m^2+c^2n^2}}$ (7 marks)

c (i) Prove that eigen value of Hermitian matrix are all real. (10 mark)

(ii) Evaluate

$\int_o^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$

By changing order of integration (10 marks)

Q4 (a) Find the eigen value & eigen vector of

$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (15 marks)

(b) Is function f is defined by

F(x, y) = $\begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

then show that f possess both partial derivative at (0, 0) but it is not continuous there at. (15 marks)

(c) Prove that equation 4x = a (l + Cos 2θ), y = b cos hϕ cosθ, z = c sin hϕ cos θ, determine hyperbolic paraboloid and angle ψ between generators through θ and ϕ is shown by

Sec ψ = $\frac{\left[(b^2+c^2)+a^4\cos^4\theta+2a^2(b^2+c^2)\cos^2\theta\cos h2\theta\right]^{1/2}}{b^2-c^2+a^2\cos^2\theta}$ (20 marks)

Section – B

Q5 (a) Solve $\frac{x\,dx + y\,dy}{x\,dy - y\,dx} = \left(\frac{1 - x^2 - y^2}{x^2 + y^2}\right)^{1/2}$ (10 marks)

(b) Find the differential equation of family of curve in xy plane passing through (-1, 1) and (1, 1)
(10 marks)

(c) Show that the orthogonal trajectory of system of confocal ellipse is self-orthogonal.
(10 marks)

(d) A particle whose mass is m, is acted upon by force m $\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at origin in time $\pi/4$.

(e) Prove that $\nabla^2 r^n$ = n(n +1) r^{n-2}, where n is constant (10 marks)

Q6 (a) Solve $(D^2 + a^2)$ y = Sec ax (15 marks)

(b) Examine whether the vector ∇u, ∇v, ∇w are coplanar where u, v, w are scalar defined by

u = x + y + z

v = $x^2 + y^2 + z^2$

w = yz + zx +xy (15 marks)

(c) A particles moves with central acceleration $\mu(r^5 - 9r)$ being projected from an apse at a distance $\sqrt{3}$ with velocity3 $\sqrt{2\mu}$ show that its path in the curve $x^4 + y^4 = 9$ (20 marks)

Q7 (a) Solve 16 $(x+1)^4 \frac{d^4y}{dx^4}$ + 96 $(x+1)^3 \frac{d^3y}{dx^3}$ + 104 $(x+1)^2 \frac{d^2y}{dx^2}$ + 8(x +1)$\frac{dy}{dx}$ + y = x^2 + 4x + 3 (15 marks)

(b) Using divergence theorem evaluate $\int\int_s A \cdot n\, d\, s$ where A = x^3i + y^3j +z^3k and S is the surface of sphere $x^2 + y^2 + z^2 = a^2$ (15 marks)

(c) If in a SHM, u, v, w be velocities at distance a, b, c from a fixed point on a straight line which is not the centre of force show that period T is given by

$\frac{4\pi^2}{T^2}$ (a – b) (b-c) (c-a) = $\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ (20 marks)

Q8 (a) Solve Cos x $\frac{d^2y}{dx^2}$ + Sin x $\frac{dy}{dx}$ – 2y Cos^3x = 2y cos^5x (10 marks)

(b) (i) Use Laplace transform to solve

$x'' - 2x' + x = e^t \left(x' = \frac{d}{dt}\right)$

Such that x(0) = 2, $x'(0) = 1$ (10 marks)

(ii) Find inverse Laplace transform of F(s) = ln $\left(\frac{s+1}{s+5}\right)$

(10 marks)

(c) Find the curvature and torsion of space curve

x = a (3u -u^3)

y = 3au^2

z = a (3u +u^3) (20 marks)