CIVIL SERVICES EXAMINATION (MAINS) 2019

DIAS PHYSICS OPTIONAL TEST SERIES

TEST 1: QUANTUM MECHANICS

Time Allowed: 3 Hours **Max Marks: 250**

Question number1is compulsory and attempt any four other questions

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Q1 (a)Quantum phenomena are often negligible in the ''macroscopic'' world. Show this numerically for the following case:The amplitude of the zero-point oscillation for a pendulum of length l = 1m and mass m = 1kg.

(b) Which of the following represent a realistic wave function of a particle?

i. $\varphi(x) = A\, e^{\frac{x^2}{a^2} + ikx}$

ii. $\varphi(x) = A\, e^{-|x|/a + ikx}$

iii. $\varphi(x) = \frac{A \sin kx}{x^2}$

iv. $\varphi(x) = A\, e^{-x/a}$ for x> o
$= A\, e^{x/a}$ for x < o

(c) Show that $\varphi\,(x) = Axe^{-x^2/2}$ is an eigen function for a linear Harmonic Oscillator of $H = \frac{-d^2}{dx^2} + x^2$. Find out energy eigen value.

(d) An electron of energy 10ev falls on a rectangular barrier of height 11 ev. Using the exact expression,find out the probability of transmission if the width is 0.1nm. Comment on the result.

(e) Whichof the following are eigen functions of L_z. For the cases where the function is an eigen function of L_z. Find the corresponding eigen value.

i. $\sin\theta\, e^{i\varphi}$

ii. $e^{i(\theta + \varphi)}$

iii. $e^{i\theta}$ Sin φ

iv. r^n Cos θ

$(10\times 5 = 50\ marks)$

Q2 (a) For a particle in a parabolic potential well find out the eigen value and eigen function in its ground state.

(b) For a simple harmonic motion of a mass attached to spring its spring constant is changed to 4 times its initial value. Find out the probability of this mass in ground state.

(c) Solve and interpret

$L^2\, Y(\theta, \emptyset) = \lambda h^2\, Y\,(\theta, \emptyset)$ (20+10+20 = 50 marks)

Q3 (a) In a 3D Cubical box find out an expression for density of state and hence or otherwise find out average energy of all electrons contained in this box (20 marks)

(b) Find out the value of $[x^2, P_x]$, $[X, L_z]$ and $[X, P_z]$ and interpret. (10 marks)

(c) Consider a system in initial state

$$\psi = \frac{A}{\sqrt{12}} \emptyset_1> + \frac{1}{\sqrt{6}} \emptyset_2> + \frac{2}{\sqrt{12}} \emptyset_3 + \frac{1}{2} \emptyset_4>$$

Find the value of A, so that it is normalized. Find out energy expectation value in this state if system is in a 1 D box of width "L" and particle of mass "m" is entrapped in it. (20 marks)

Q4 (a) For Pauli Spin matrices show that $\vec{\sigma} \times \vec{\sigma} = 2i\vec{\sigma}$. Interpret the result. (10 marks)

(b) Formulate the Schrodinger's wave equation for hydrogen atom and derive an expression for energy eigen values. Draw the graphs of wave functions in first three states. (5+20+5 = 30marks)

(c) Estimate the de Broglie Wave length of the electron of energy 1 MeV. (10 marks)

Q5 (a) Solve the Schrödinger equation for a step potential and calculate the transmission and reflection coefficient for the case when the kinetic energy of the particle E_0 is greater than the potential energy V (i.e., $E_0 > V$). (20 marks)

(b) An electron is confined to move between two rigid walls separated by 10^{-9} m. Compute the de Broglie wavelengths representing the first three allowed energy states of the electron and the corresponding energies. (10 marks)

(c) Using uncertainty principle, calculate the size and energy of the ground state of hydrogen atom. (10 marks)

(d) For a hydrogen atom in ground state find out the most probable distance and expectation value of r^2. (10 marks)

Q6 (a) Nuclear forces are mediated by exchange of $\pi - mesons$ of rest mass 140MeV. Using Heisenberg's uncertainty principle, estimate the range of nuclear forces. (10 marks)

(b) The wave function of an electron at an instant is given by $\psi = f(r, \theta)\, e^{2i\emptyset} \chi_{1/2}$. Calculate the average value of the z-component of its magnetic moment. (10 marks)

(c) Find the eigenvalues and spin eigenstates of the operator $S_1 = S_x + S_y$. (10 marks)

(d) Prove Heisenberg's uncertainity principle

$$\Delta \times \Delta p_x \geq \hbar/2$$

Hence or otherwise show that minimum value of product occurs for a Gaussian wave packet (20 marks)