CIVIL SERVICES EXAMINATION (MAINS)

Paper 1: Linear Algebra

TUTORIAL SHEET 1: Vector Space

1. If $w_1 = \{(x, y, z)|\ x + y - z = 0\}$
   $w_2 = \{(x, y, z)|3x + y - 2z = 0\}$
   $w_3 = \{(x, y, z)|x - 7y + 3z = 0\}$
   Find dim $(w_1 \cap w_2 \ \cap w_3)$ and dim $(w_1 + w_2)$ **[2016]**

2. Suppose U and W are distinct four dimensional sub space of a vector space V, where dim V=6. Find the possible dimension of subspace $U \cap W$. **[2017]**

3. Express basis vector $e_1 = (1, 0)$ and $e_2 = (0,1)$ as linear combination of $\alpha_1 = (2, -1)$ and $\alpha_2 = (1,3)$ **[2018]**

4. Let A = $\begin{pmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{pmatrix}$

   i. Find the rank of matrix A
   ii. Find dimension of subspace

   $$V = \left\{ (x_1,\ x_2, x_3, x_4) \in R^4 \middle| A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}$$ **[2019]**

TUTORIAL SHEET 2: Linear Transformation

1. Show that $f: R^3 \to R$ is a linear transformation, where $f(x,y,z) = 3x + y - z$. What is the dimension of the Kernel? Find a basis for the Kernel. **[2004]**

2. Show that the linear transformation from R^3 to R^4 which is represented by the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is one to one. Find a basis for its image. **[2004]**

3. Let T be a linear transformation on R^3 whose matrix relative to the standard basis of R^3 is $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. Find the matrix of T relative to the basis $\beta\{(1,1,1),(1,1,0),(0,1,1)\}$

[2005]

4. If $T: R^2 \to R^2$ is defined by $T(x,y) = (2x-3y,\ x+y)$, compute the matrix of T relative to the basis $\beta = \{(1,2),(2,3)\}$. **[2006]**

5. Let T be the linear transformation from R^3 to R^4 defined by

$$T(x_1,x_2,x_3) = (2x_1 + x_2 + x_3, x_1 + x_3, 3x_1 + x_2 - 2x_3) \text{ for every } (x_3,x_2,x_1) \in R^3.$$

Determine the basis for the null space of T. What is the dimension of the Range space T?

[2007]

6. Consider the vector space $X = \{p(x)\}$ is a polynomial of degree less than or equal to 3 with real coefficients over the field of □ . Define the map $D: X \to X$ by

$$(DP)(x) = P_1 + 2P_2x + 3P_3x^2 \text{ where } P(x) = P_0 + P_1x + P_2x^2 + P_3x^3.$$

Is D a linear transformation on X? If it is then construct the matrix representation for D with respect to the ordered basis $\{1, x, x^2, x^3\}$ for X. **[2007]**

7. Show that $B=\{(1,0,0),(1,1,0),(1,1,1)\}$ is a basis for R^3. Let $T:R^3 \to R^3$ be a linear transformation s.t

$$T(1,0,0)=(1,0,0)$$

$$T(1,1,0)=(1,1,1)$$

$$T(1,1,1)=(1,1,0)$$

Find $T(x,y,z)$. **[2008]**

8. Let $\beta=\{(1,1,0),(1,0,1),(0,1,1)\}$ and $\beta'=\{(2,1,1),(1,2,1),(-1,1,1)\}$ be the two ordered basis of R^3. Then find a matrix representing the linear transformation $T:R^3 \to R^3$ which transform β into β'. Use this matrix representation to find $T(X)$, where $X=(2,3,1)$.
[2009]

9. Let $L:R^4 \to R^3$ be a linear transformation defined by
$L(x_1,x_2,x_3,x_4)=(x_3+x_4-x_1-x_2,x_3-x_2,x_4-x_1)$. Then find the rank and nullify of L. Also, determine the null space and the range space of L. **[2009]**

10. What is the null space of the differentiation transformation $\frac{d}{dx}:P_n \to P_n$, where P_n is the space of all polynomials of degree $\le n$ over the real numbers? What is the null space of the second derivative as a transformation of P_n? What is the null space of the K^{th} derivative of P_n? **[2010]**

11. Let $M=\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$. Find the unique linear transformation $T:R^3 \to R^2$ so that M is the matrix of T with respect to the basis $\beta=\{V_1=(1,0,0),V_2=(1,1,0),V_3=(1,1,1)\}$ of R^3 and $\beta'=\{\omega_1=(1,0),\omega_2=(1,1)\}$ of R^2. Also find $T(x,y,z)$. **[2010]**

12. (a) In space of R^n determine whether or not the set $\{e_1-e_2,e_2-e_3,....e_{n-1}-e_n,e_n-e_1\}$ is linearly independent.
(b) Let T be a linear transformation from a vector space V over real into V s.t. $T-T^2=I$. Show that T is invertible. **[2010]**

13. Find the nullity and a basis of the null space of the linear transformation $A: R^4 \to R^4$ given by the matrix

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ **[2011]**

14. (i) Show that the vectors $(1,1,1),(2,1,2)$ and $(1,2,3)$ are linearly independent in R^3. Let $T: R^3 \to R^3$ be a linear transformation defined by

$$T(x,y,z) = (x+2y+3z, x+2y+5z, 2x+4y+6z).$$

Show that the image of above vectors under T are linearly independent. Give the reason for the same. **[2011]**

15. Let $T: R^3 \to R^3$ be the linear transformation defined by

$$T(\alpha,\beta,\gamma) = (\alpha+2\beta-2\gamma, 2\alpha+5\beta-4\gamma, \alpha+4\beta+\gamma).$$

Find the basis and the dimension of the image of T and the kernel of T.

[2012]

16. Consider the linear mapping $f: R^2 \to R^2$ by $f(x,y) = (3x+4y, 2x-5y)$. Find the matrix A relative to the basis $(1,0),(0,1)$ and matrix B relative to the basis $(1,2)(2,3)$.

[2012]

17. Let P_n denote the vector space of all real polynomials of degree at most n and $T: P_2 \to P_3$ be linear transformation given by $T(f(x)) = \int_0^x P(t)\,dt$, $P(x) \in P_2$. Find the matrix of T with respect to the basis $\{1, x, x^2\}$ and $\{1, x, 1+x^2, 1+x^3\}$ of P_2 and P_3 respectively. Also find the null space of T. **[2013]**

18. Let V be an n- dimensional vector space and $T:V \to V$ be an invertible linear operator. If $\beta = \{X_1, X_2, ...X_n\}$ is a basis for V, show that $\beta' = \{TX_1, TX_2, ...TX_n\}$ is also a basis of V. **[2013]**

19. Let P_n denote the vector space of all real polynomials of degree at most n and $T:P_2 \to P_3$ be a linear transformation given by

$$T(p(x)) = \int_0^x p(t)\,dt \qquad p(x) \in P_2$$

Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1+x^2, 1+x^3\}$ of P_2 and P_3 respectively. Also, find the null space of T. **[2014]**

20. Let V be an n- dimensional vector space and $T:V \to V$ be an invertible linear operator of $\beta = \{X_1, X_2....X_n\}$ is a basis for V. Show that $\beta' = \{TX_1, TX_2....X_n\}$ is also a basis of V. **[2014]**

21. Let $V = R^3$ and $T \in A(V)$ for all $a_i \in A(V)$ be defined by

$$T(a_1, a_2, a_3) = (2a_1 + 5a_2 + a_3, -3a_1 + a_2 - a_3, -a_1 + 2a_2).$$

What is the matrix of T relative to the basis

$V_1 = (1,0,1),\ V_2 = (-1,2,1),\ V_3 = (3,-1,1)$ **[2015]**

22. If $M_2(R)$ is space of real matrices of order 2×2 and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of $T: M_2(R) \to P_2(x)$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + c + (a-d)x + (b+c)x^2$$

with respect to the standard basis of $M_2(R)$ and $P_2(x)$. Further find the null space of T.

[2016]

23. If $T: P_2(x) \to P_3(x)$ is s.t. $T(f(x)) = f(x) + 5\int_0^x f(t)\,dt$, then choosing $\{1, 1+x, 1-x^2\}$ and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ respectively, find the matrix of T.

[2016]

24. Of $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation $T: P_2(x) \to P_2(x)$ with respect to the bases $\{1-x,\ x(1-x),\ x(1+x)\}$ and $\{1, 1+x, 1+x^2\}$, then find T. **[2016]**

25. Consider the matrix mapping $A: R^4 \to R^3$ where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Find the basis and dimension of the image of A and those of the kernel A. **[2017]**

26. Let T : $R^2 \to R^2$ be a linear map such that T(2, 1) = (5, 7) and T(1 ,2)= (3, 3). If A is the matrix corresponding of T with respect to standard bases e_1, e_2 then find rank of A.
[2019]

Linear Algebra

TUTORIAL SHEET 3: Matrix Algebra

1. Find the inverse of the matrix given below using elementary row operations only

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$ **[2005]**

2. Using elementary row operations, find the rank of the matrix

$$\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$ **[2006]**

3. Show that the matrix A is invertible if and only if the $adj(A)$ is invertible. Hence find $|adj(A)|$. **[2008]**

4. Let A be a non-singular $n\times n$ square matrix. Show that $A(adjA)=|A|I_n$. Hence show that $|adj(adjA)|=|A|^{(n-1)^2}$. **[2011]**

5. Find the rank of the matrix

$$A=\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 5 & 8 & 12 & 17 \\ 3 & 5 & 8 & 17 & 23 \\ 8 & 12 & 17 & 23 & 30 \end{bmatrix}.$$ **[2014]**

6. Find the inverse of the matrix

$$A=\begin{bmatrix}1 & 3 & -1\\ 2 & -1 & 7\\ 3 & 2 & -1\end{bmatrix}$$

By using elementary row operations. Hence solve the system of linear equations

$$x+3y+z=10$$

$$2x-y+7z=21$$

$$3x+2y-z=4$$

[2014]

7. Find the rank of the matrix

$$A=\begin{bmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 3 & 5 & 8 & 12\\ 3 & 5 & 8 & 12 & 17\\ 3 & 5 & 8 & 17 & 23\\ 8 & 12 & 17 & 23 & 30\end{bmatrix}.$$

[2014]

8. Reduce the following matrix to row echelon form and hence find the rank.

$$\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 5\\ 1 & 5 & 5 & 7\\ 8 & 1 & 14 & 17\end{bmatrix}.$$

[2015]

9. Using elementary row operations, find the inverse of $A=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 2\\ 1 & 0 & 1\end{bmatrix}$

[2016]

10. Let A be a 3×2 matrix and B a 2×3 matrix. Show that $C=A\cdot B$ is singular matrix.

[2018]

Linear Algebra

TUTORIAL SHEET 4: Solution of System of Linear Equation

1. Verify whether the following system of equation is consistent

$$x+3z=5$$
$$-2x+5y-z=0$$
$$-x+4y+z=4$$

[2004]

2. Investigate for what values of λ and μ are equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

have

(i) no solution

(ii) a unique solution

(iii) infinitely many solutions **[2006]**

3. Let $A=\begin{bmatrix}1 & 0 & -1\\ 3 & 4 & 5\\ 0 & 6 & 7\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $B=\begin{bmatrix}2\\ 6\\ 5\end{bmatrix}$

Solve the system of equations given by $AX=B$. Also solve the system of equations $A^T X=B$, where A^T denotes the transpose of the matrix A. **[2011]**

4. Find the dimension and a basis for the space w of all solutions of the following homogenous system using matrix notation

$$x_1+2x_2+3x_3-2x_4+4x_5=0$$
$$2x_1+4x_2+6x_3+x_4+9x_5=0$$
$$3x_1+6x_2+13x_3+4x_4+14x_5=0$$

[2012]

5. Find the inverse of the matrix

$$A=\begin{bmatrix}1 & 3 & 1\\ 2 & -1 & 7\\ 3 & 2 & -1\end{bmatrix}$$

by using elementary row operations. Hence solve the system of linear equations

$$x+3y+z=10$$

$$2x-y+7z=21$$

$$3x+2y-z=4$$

[2014]

6. Using elementary row operations find the condition that the Linear equations

$$x-2y+z=a$$
$$2x+7y-3z=b$$
$$3x+5y-2z=c$$

have a solution. **[2016]**

7. Consider the following system of equations in x, y, z

$$x+2y+2z=1$$
$$x+ay+3z=3$$
$$x+11y+az=b$$

(i) for which values of a does the system have a unique solution?

(ii) For which pair of values (a,b) does the system have more than one solution?

[2017]

8. For the system of Linear equations

$$x+3y-2z=-1$$
$$5y+3z=-8$$
$$x-2y-5z=7$$

determine which of the following statements are true and which are false

(i) The system has no solution

(ii) The system has a unique solution

(iii) The system has infinitely many solutions **[2018]**

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9. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$ then show that $AB=6I_3$. Use the result to solve the system of equation **[2019]**

$$2x + y + z = 5$$
$$x - y = 0$$
$$2x + y - z = 1$$

TUTORIAL SHEET 5: Eigen Value Problem

1. Find the characteristic polynomial of the matrix $A=\begin{bmatrix}1 & 1\\ -1 & 3\end{bmatrix}$. Hence find A^{-1} and A^{6}

[2004]

2. If S is a skew-Hermitian matrix, then show that it is a unitary matrix

also show that if $A=(I+S)(I-S)^{-1}$ every unitary matrix can be expressed in the above

from provided -1 is not an eigenvalue of A

[2005]

3. State Cayley-Hamilton theorem and using it, find the inverse of $\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$ **[2006]**

4. Let A be a non-singular matrix. Show that if $I+A+A^2+...+A^n=0$ then $A^{-1}=A^n$

[2008]

5. Find a hermitian and skew Hermitian matrix each of whose sum is the matrix

$$\begin{bmatrix}2i & 3 & -1\\ 1 & 2+3i & 2\\ -i+1 & 4 & 5i\end{bmatrix}$$

[2009]

6. If μ_1 , μ_2 , μ_3 are the eigen values of the matrix $A=\begin{bmatrix}26 & -2 & 2\\ 2 & 21 & 4\\ 44 & 2 & 28\end{bmatrix}$ show that $(\mu_1{}^2+\mu_2{}^2+\mu_3{}^2)^{1/2} \leq \sqrt{1949}$

[2010]

7. Find a 2×2 real matrix A which is both orthogonal and skew-symmetric can these exist a 3×3 real matrix which is both orthogonal and skew-symmetric? Justify your answer.

8. Let A and B be $n\times n$ matrices over reals show that $I-BA$ is invertible if $I-AB$ is invertible. Deduce that AB and BA have the same eigenvalues

9. Let $\lambda_1, \lambda_2, ..., \lambda_n$ be the eigenvalues of a $n \times n$ square matrix A with corresponding eigen vectors $x_1, x_2, ..., x_n$. If B is a matrix similar to A. Show that the eigenvalues of B are same as that of A. Also find the relation between the eigen vectors of A. Also find the relation between the eigen vectors of B and eigen vectors of A. **[2011]**

10. Verify the Cayley-Hamilton theorem for the matrix.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}$$

Using this show that A is non singular and find A^{-1}

11. Let $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be non-singular matrix of order 3×3. Find the eigen values of the matrix B^3 where $B = C^{-1}AC$ **[2011]**

12. If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of $Adj\, A$ **[2012]**

13. Let $H = \begin{bmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{bmatrix}$ be a Hermitian matrix . find a non-singular matrix P such that $D = P^{+}H\bar{P}$ is a diagonal. **[2012]**

14. Let A be a Hermitian matrix having all distinct eigen values $\lambda_1, \lambda_2, ..., \lambda_n$. If $x_1, x_2, ..., x_n$ are corresponding eigen vectors then show that the $n \times n$ matrix C where K^{th} column consists of the vector X_K is non singular **[2013]**

15. Let $A=\begin{bmatrix}1 & 1 & 1\\ 1 & \omega^2 & \omega\\ 1 & \omega & \omega^2\end{bmatrix}$ where $\omega(\neq 1)$ is a cube root of unity of $|\lambda_1|+|\lambda_2|+|\lambda_3|\leq 9$

[2013]

16. Let A be a square matrix and A^* be its adjoint, show that the eigen values of matrices AA^* and A^*A are real further show that $trace(AA^*)=trace(A^*A)$

17. Prove that the eigen values of a unitary matrix have absolute value 1. **[2014]**

18. Verify Cauchy-Hamilton theorem far the matrix $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ and hence find the inverse . also, find the matrix represented by $A^5-4A^4-7A^3+1A^2-A-10I$ **[2014]**

19. Let $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & 2 & 0\end{bmatrix}$. Find the eigen values of A and the corresponding eigen vector. **[2014]**

20. If matrix $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$ then find A^{30} **[2015]**

21. Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$$ **[2015]**

22. If $A=\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$, then find $A^{14}+3A-2I$ **[2016]**

23. If $A=\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, then find the eigenvalues and eigenvectors of A **[2016]**

24. Prove that eigenvalues of Hermitian matrix are real **[2016]**

25. Let $A=\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}$. Find the non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix. **[2017]**

26. Show that similar matrices have the some characteristic polynomial. **[2017]**

27. Prove that distinct non-zero eigenvectors of a matrix are linear independent. **[2018]**

28. Show that of A and B are similar $n\times n$ matrices, then they have the some eigenvalues. **[2018]**

29. Let A and B be two orthogonal matrices of same order and $det\, A + det\, B = 0$. Show that A+B is a singular matrix. **[2019]**

30. State Cayley Hamilton theorem, use the theorem to find A^{100}, where

$$A=\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$ **[2019]**

MATHEMATICS PAPER I: Calculus

TUTORIAL SHEET 6: Function of Real Variables, Limit and Indeterminate Form, Function of two or three variables

Q1. Find value of a & b such that
$\lim_{x \to 0} \frac{a\sin^2 x + b \log \cos x}{x^4} = \frac{1}{2}$ **[2006]**

Q2. $\lim_{x \to 1} \ln(1-x) \cot \frac{\pi x}{2}$ **[2008]**

Q3. Find $\lim_{(x,y) \to (0,0)} \frac{x^2 y}{x^3+y^3}$ if it exist **[2011]**

Q4. Evaluate $\lim_{x \to 2} f(x)$ where $f(x) = \begin{cases} \frac{x^2-9}{x-2} & ; x \neq 2 \\ \pi & ; \quad 2 \end{cases}$ **[2011]**

Q5. Define sequence S_n of real numbers by
$S_n = \sum_{i=1}^{n} \frac{(\log(n+i) - \log n)^2}{n+i}$
Does limit $\underset{n \to \infty}{limit} \; S_n$ exist if so find it.

Q6. $\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ **[2015]**

Q7. Find $\lim_{n \to \infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}$ **[2018]**

Q8. Determine if $\lim_{z \to 1} (1-z) \tan \frac{\pi z}{2}$ exist or not.
If the limit exist then find the value. **[2018]**

Q9. Let $f : \left[0, \frac{\pi}{2}\right] \to R$ be continuous function such that $f(x) = \frac{\cos^2 x}{4x^2 - \pi^2}$, $0 \leq x < \frac{\pi}{2}$ find value of $f\left(\frac{\pi}{2}\right)$ **[2019]**

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TUTORIAL SHEET: 7 Continuity and Differentiability of two and three variables

Q1. Let $f = R^2 \to R$ be defined as
$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$, $(x,y) \neq (0,0) f(0,0) = 0$
Prove that $f_x and\ f_y$ exist at (0, 0) but f is not differentiable at (0, 0) **[2005]**

Q2. Show that function given below is not continuous at origin
$f(x,y) = \{^{0\ if\ xy=0}_{1\ if\ xy\neq 0}$ **[2005]**

Q3. Find a & b so that $f'(2)$ exist
Where $f(x)\ \{^{\frac{1}{|x|}, if\ |x|>2}_{a+bx^2, if\ |x|\leq 2}$ **[2006]**

Q4. Let $f(x)$ $(x \in (-\pi,\pi))$ be defined by
$f(x) = Sin|x|$. If f is continuousfx $(-\pi,\pi)$. If it is continuous, then is it differentiable on $(-\pi,\pi)$ **[2007]**

Q5. Let f = $R^2 \to R$ be defined
$$F(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, if\ (x,y) \neq (0,0) \\ 0\ if\ (x,y) = (0,0) \end{cases}$$
Is f continuous at (0, 0) compute partial derivative f at any point (x,y) if it exist
[2009]

Q6. Let f be function defined in R
Such that $f(0)$= -3 , $f'(x) \leq 5$ for all values of x and R
Can f(2) possibly be?

Q7. Let p & q be positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$. Show that for real number
$a, b\ \geq 0$
$$ab \leq \frac{a^p}{p}+\frac{b^q}{q}$$

Q8. Define a function f of two real variables in the xy plane by
$$\begin{cases} \frac{x^3 cos\frac{1}{y} + y^3 cos\frac{1}{x}}{x^2+y^2} & for\ x,y \neq 0 \\ 0, otherwise \end{cases}$$
check continuity & differentiability of f at (0, 0)
[2012]

Q9. For the function

$$f(x,y) = \begin{cases} \frac{x^2 - x\sqrt{y}}{x^2 + y}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Examine the continuity & differentiability **[2015]**

Q10. Let $f(x,y) = \begin{cases} \frac{2x^4y - 5x^2y^2 + y^5}{(x^2+y^2)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Find a $\delta > 0$ such that $|f(x,y) - f(0,0)| < 0.1$ whenever $\sqrt{x^2 + y^2} < \delta$ **[2016]**

Q11. Let $f : D(\leq R^2) \rightarrow R$ be a function and $[a, b] \in D$. If $f(x,y)$ is continuous at (a, b) then show that the function $f(x, b)$ and $f(a, y)$ are continuous at $x = a$ and $y = b$ respectively. **[2019]**

Q12. Is f(x) = $|cos\, x| + |sin\, x|$ differentiable at x = $\frac{\pi}{2}$. If yes, find its derivative at x = $\frac{\pi}{2}$. If no, then give a proof of it **[2019]**

TUTORIAL SHEET 8: Mean Value Theorem, Taylors Theorem's With Remainders

Q1. Use the mean value theorem to prove that

$$\frac{2}{7} < \log 1.4 < \frac{2}{5}$$ **[2000]**

Q2.Show that

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}, x > 0$$ **[2004]**

Q3. If f' and g' exist for every x $\epsilon [a, b]$ and if $g'(x)$ does not vanish anywhere in (a,b) , show that there exists c in (a, b) such that

$$\frac{f(c)-f(a)}{g(b)-g(c)} = \frac{f'(c)}{g'(c)}$$ **[2005]**

Q4. If f is derivative of some function defined on [a, b] prove that there exist a number $n \in [a, b]$ & $\int_a^b f(t)dt = f(n)(b-a)$ **[2009]**

Q5. Suppose that f'' is continuous on [1, 2] and that f has three zeroes in the interval (1, 2). Show that f'' has atleast one zero in the interval (1, 2) **[2009]**

Q6. A three differentiable function $f(x)$ is such that $f(a) = 0 \ = f(b)$ & $f(c) > 0$ for $a < c < b$. Prove that there be at least one point $\in$, $a < \in < b$, for which $f''(\in) < 0$ **[2010]**

Q7. Prove that between two real roots of $e^x \cos x + 1 = 0$ a real root of $e^x \sin x + 1 = 0$ **[2014]**

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TUTORIAL SHEET 9: Partial Derivatives/Jacobians

Q1. If $x\cos u + y\sin u = 1$;

$v = x\sin u - y\cos u$

Prove that

$$v^2\frac{\partial^2 u}{\partial x\partial y} + v\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} = \cos 2u$$

[1982]

Q2.Show that under the transformation

$u = x^2 - y^2$, $v = xy$, the equation $y^2\frac{\partial^2 H}{\partial x^2} - x^2\frac{\partial^2 H}{\partial y^2} = x\frac{\partial H}{\partial x} - y\frac{\partial H}{\partial y}$ becomes

$$\left(u\frac{\partial}{\partial v} - v\frac{\partial}{\partial u}\right)\frac{\partial H}{\partial v} = 0$$

[1983]

Q3. Obtain a set of sufficient condition such that for a function f(x, y)

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$$

[1983]

Q4. $If\ u = \frac{(ax^3 + by^3)^n}{3n(3n-1)} + x\,f(\frac{y}{x})$

Find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

[1986]

Q5. $If\ u = cosec^{-1}\left(\frac{x^{\frac{1}{n}+1} + y^{\frac{1}{n}+1}}{x+y}\right)^{1/2}$ then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{4n^2}\tan u\,(2n + sec^2 u)$$

[1987]

Q6. $If\ x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$ then prove that

$$(dx)^2 + (dy)^2 + (dz)^2 = (dr)^2 + r^2(d\theta)^2 + r^2 sin^2\theta(d\varphi)^2$$

[1992]

Q7. If $u = f\left(\frac{x}{a}, \frac{y}{b}, \frac{z}{c}\right)$ prove that
$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ **[1996]**

Q8. If $z = x\, f\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ show that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$ **[2006]**

Q9. Prove that if $z = \phi(y + ax) + \psi(y - ax)$
Then $a^2\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2} = 0$ for any twice differential ϕ and ψ, a is constant **[2007]**

Q10. If f(x, y) is homogeneous function of degree n in x and y, and has continuous first and second order partial derivative then show that

(1) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

(2) $x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} = n(n-1)f$ **[2010]**

Q11. If $f(x,y) = \begin{cases} xy\frac{(x^2-y^2)}{x^2+y^2} & ; (x,y) \neq (0,0) \\ 0 \ , & (x,y) = (0,0) \end{cases}$

Calculate $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ at (0, 0) **[2011]**

Q12. Compute $f_{xy}(0,0)$ & $f_{yx}(0,0)$ for function

$$f(x,y) = \begin{cases} \frac{xy^3}{x+y^2}, & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Also discuss continuity of f_{xy}, f_{yx} at (0, 0) **[2013]**

Q13. Let $f(x,y) = xy^2 \ if \ y > 0$
$= -xy^2 if \ y \leq 0$
Determine which of $\frac{\partial f}{\partial x}(0,1)$ & $\frac{\partial f}{\partial y}(0,1)$ exist & which does not exist. **[2018]**

Q14. If $u = sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $sin^2 u$ is a homogeneous function of n & y degree $-\frac{1}{6}$ hence show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{tan^2 u}{12}\right)$
[2019]

Jacobian

Q1. $If\ u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$ find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v related? If so, find the relationship **[1984]**

Q2. $If\ u = x + y - z\ , v = x - y + z\ , w = x^2 + (y - z)^2$ Examine whether or not there exists any functional relationship between u , v, w and find the relation if any **[1987]**

Q3. If roots of equation $(\lambda - u)^3 + (\lambda - v)^3 + (\lambda - w)^3 = 0$ in λ are x,y,z , show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = -\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$ **[2004] [2006]**

Q4. If u, v, w are roots of equation in λ and $\frac{x}{a+\lambda} + \frac{y}{b+\lambda} + \frac{z}{c+\lambda} = 1$ evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ **[2005]**

Q5. If $u = x + y + z,\ \ uv = y + z, uvw = z$, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ **[2005]**

Q6. Using, Jacobian method, show that if $f'(x) = \frac{1}{1+x^2}$ and f(0)=0, then $f(x) + f(y) = f(\frac{x+y}{1-xy})$ **[2019]**

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TUTORIAL SHEET 10: Maxima/minima, Lagrange's method of multipliers

Q1. Find a rectangle parallelepiped of greatest volume for a given total surface area S, using lagrange's method of multiplier. **[2007]**

Q2. A space probe in the shape of ellipsoid $4x^2 + y^2 + z^2 = 16$ enters the earth's atmosphere and its surface begin to heat. After one hour temperature, at the point (x, y, z) on the probe surface is given by
$T(x, y, z) = 8x^2 + 4yz - 16z + 600$
Find the hottest point on the probe surface. **[2009]**

Q3. Show that a box (rectangular parallelepiped) of maximum volume v with prescribed surface area is a cube. **[2010]**

Q4. Find point on sphere $x^2 + y^2 + z^2 = 4$ that are closest to & farthest from the point (3, 1, -1)
[2011]

Q5. Find the point of local extreme and saddle point of function f of two variables defined by
$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ **[2012]**

Q6. Using lagrange's multiplier method to find shortest distance between the line $y = 10 - 2x$ and ellipse
$\frac{x^2}{4} + \frac{y^2}{9} = 1$ **[2013]**

Q7. Find maximum or minimum value of $x^2 + y^2 + z^2$ suggest to conditum $ax^2 + by^2 + cz^2 = 1$ & $lx + my + nz = 0$ interpret result geometrically.
[2014]

Q8. Find the height of cylinder of maximum volume that can be inscribed in a sphere of radius a. **[2014]**

Q9. Which point of sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from point (2, 1, 3). **[2015]**

Q10. A conical tent is of given capacity. For the least amount of canvas required for it. Find the ratio of its height to the radius of its base. **[2015]**

Q11. Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the condition $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $x + y - z = 0$ **[2016]**

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Q12. Find the maximum and minimum value of $x^4 - 5x^2 + 4$ in interval [2, 3]
[2018]

Q13. Find the shortest distance from point (1, 0) to the parabola $y^2 = 4x$
[2018]

Q14. Find the maximum and minimum value of function $f(x) = 2x^3 - 9x^2 + 12x + 6$ on interval [2, 3] **[2019]**

TUTORIAL SHEET 11: Asymptotes & Curve Tracing

Q1. Find the asymptotes of the cubic

$x^3 - xy^2 - 2xy + 2x - y = 0$ and show that they cut the curve again in points which lie on the line 3x-y =0. **[1988]**

Q2. Sketch the curve

$(x^2 - a^2)(y^2 - b^2) = a^2b^2$ **[1991]**

Q3.Find the cubic curve which has the same asymptotes as the curve

$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$ and which passes through the points (0, 0) ,(1,0) and (0,1) **[1991]**

Q4. Find the asymptotes of the curve

$4(x^4 + y^4) - 17x^2y^2 - 4x(4y^2 - x^2) + 2(x^2 - 2) = 0$ and that they pass through the point of intersection of the curve with the ellipse

$(x^2 + 4y^2) = 4$. **[1996]**

Q5.Show that the asymptotes of the curve

$(x^2 - y^2)(y^2 - 4x^2) + 6x^3 - 5x^2y - 3xy^2 + 2y^3 - x^2 + 3xy - 1 = 0$

Cut the curve again in eight points which lie on a circle of radius 1. **[1997]**

Q6. Find the asymptotes of the curve

$(2x - 3y + 1)^2(x + y) - 8x + 2y - 9 = 0$

and show that they intersect the curve again in three points which lie on the straight line.

[1998]

Q7. Find three asymptotes of the curve

$x^3 + 2x^2y - 4xy - 8y^3 - 4x + 8y - 10 = 0$.

Also find the intercept of one asymptotes between the other. **[1999]**

Q8. Find the equation of the cubic curve which has the same asymptotes as

$2x(y-3)^2 = 3y(x-1)(x-1)^2$

and which touches the x axis at the origin and passes through the point (1, 1)

[2001]

TUTORIAL SHEET 12:Indifinite Integral and Riemann's definition of definite integral

Q1. $\int_0^1 (x ln x)^3 dx$ **[2008]**

Q2. Does $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, dx$ exist, if to find its value **[2010]**

Q3. $\int_0^1 ln x\ dx$ **[2011]**

Q4. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ **[2014]**

Q5. $\int_{\pi/6}^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx$ **[2015]**

TUTORIAL SHEET 13: Infinite and Improper Integral

Q1. $\textit{Prove that } \Gamma(n)\Gamma(n+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2n-1}}$ where n > 0 **[1997]**

Q2. Test convergence if $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}\, dx$ **[2001]**

Q3. Test convergent of

(1) $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}$

(2) $\int_0^\infty \frac{\sin^2 x}{x} dx$ **[2003]**

Q4.Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}$ dx **[2005]**

Q5.Express $\int_0^1 x^m (1-x^n)^p dx$ in terms of gamma function & hence evaluate the integral $\int_0^1 x^6 \sqrt{1-x^2}\, dx$ **[2006]**

Q6. Show that $e^{-x}x^n$ is bounded on $[0,\infty]$ for all positive integrals value of n using this result show that $\int_0^\infty e^{-x}x^n dx$ **[2007]**

Q7. Find all the real values of p & q so that integral $\int_0^1 x^p \left(\log\frac{1}{x}\right)^q dx$ **[2012]**

Q8. $\int_0^1 \left(2x \sin\frac{1}{x} - \cos\frac{1}{x}\right) dx$ **[2013]**

Q9. Evaluate I = $\int_0^1 \sqrt[3]{x \log\left(\frac{1}{x}\right)}\, dx$ **[2016]**

Q10. Examine if the improper integral $\int_0^3 \frac{2x\, dx}{(1-x^2)^{2/3}}$ exists. **[2017]**

TUTORIAL SHEET 14: Double and Triple Integral

Q1. Show that $\iint x^{m-1}y^{n-1}dx$ over position quadrant of ellipse $\frac{x^2}{a}+\frac{y^2}{b}=1$ is $\frac{a^m b^n}{4}\frac{\Gamma\frac{m}{2}\Gamma\frac{n}{2}}{\Gamma\frac{m}{2}+\frac{n}{2}+1}$ **[1999]**

Q2. Show $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy=\frac{\pi}{4}$ **[2002]**

Q3. $\int_0^a\int_{y^2/a}^{y}\frac{y\,dx\,dy}{(a-x)\sqrt{ax-y^2}}$ **[2003]**

Q4. Change order of integration of
$\int_0^\infty\int_0^\infty\frac{e^{-y}}{y}\,dy\,dx$ & hence evaluate it **[2006]**

Q5. Evaluate $\int_0^a\int_y^a\frac{x\,dx\,dy}{x^2+y^2}$ by changing order integration **[2008]**

Q6. Let D be the region determine by inequalities
$x>0, y>0, z>8$ & $z>x^2+y^2$ compute
$\iint_D 2x\,dx\,dy\,dz$ **[2010]**

Q7. Evaluate $\iint_D xy\,dA$ where D is bounded by
$y=x-1$ &parabola $y^2=2x+6$

Q8. $\iint\{(xy(1-x-y))\}^{1/2}\,dx\,dy$ over $x=0, y=0$, $x+y=1$ by using $x+y=u$, $y=uv$ **[2014]**

Q9. $\iint_R\sqrt{|y-x|^2}\,dx\,dy$ where $R=[-1,1;\ 0,2]$ **[2015]**

Q10. Evaluate the integral $\iint_R(x-y)^2\cos^2(x+y)dx\,dy$ when R is rhombus with vertices $(\pi,0),(2\pi,\pi)\ (\pi,2\pi),(0,\pi)$
[2015]

Q11. Evaluate $\int_0^a\int_{x/a}^{a}\frac{x\,dy\,dx}{x^2+y^2}$ **[2016]**

Q12. Evaluate $\iint_R f(x,y)\,dx\,dy$ over the rectangle R = [0, 1; 0,1] where
$f(x,y)=\begin{cases}x+y, if\ x^2<y<2x^2\\ 0\qquad,\quad elsewhere\end{cases}$ **[2016]**

Q13. Integrate the function $f(x,y) = xy(x^2 + y^2)$ are domain $R:\{-3 \le x^2 - y^2 \le 3,\ 1 \le xy \le 4\}$ **[2017]**

Q14. Prove that $\frac{\pi}{3} \le \iint_D \frac{dxdy}{\sqrt{x^2+(y-2)^2}} \le \pi$ where D is the unit disc **[2017]**

TUTORIAL SHEET 15: Area, Surface, Volume

Q1. Find volume of solid generated by revolving the cardioide $r = a(1 - \cos\theta)$ about initial line. **[2001]**

Q2. Evaluate $\iiint (x+y+z+1)^2 dx\, dy\, dz$ over
$x \geq 0, \quad y \geq 0, \quad z \geq 0\,, \quad x+y+z \leq 1$ **[2002]**

Q3. Find the volume generated by revolving bounded by curve
$(x^2 + 4a^2)y = 8a^3,\ 2y = x\ \&\ x = 0$ about y axis **[2003]**

Q4. Evaluate $\iiint_v z\, dv$ where V is volume bounded by cone $x^2 + y^2 = z^2$ lying on positive side of y axis.
[2005]

Q5. Find volume of uniform ellipsoid $\frac{x^2}{a} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **[2006]**

Q6. Obtain volume bounded by elliptic paraboloid given by $z = x^2 + 9y^2$ &
$z = 18 - x^2 - 9y^2$ **[2008]**

Q7. Evaluate I = $\iint_s x\, dy\, dz + dz\, dx + xz^2 dx\, dy$ where S is outer side of part of sphere $x^2 + y^2 + z^2 = 1$ in first octant. **[2009]**

Q8. Prove $\int \frac{x^2+y^2}{p} dx = \frac{\pi ab}{4}[4 + (a^2 + b^2)(a^{-2} + b^{-2})]$ where integral is taken round the ellipse
$\frac{x^2}{a^2} + \frac{y^2}{b^2}$=1 & p is three length of three perpendicular from centre to tangent.**[2009]**

Q9. Find surface area of plane $x + 2y + 2z = 12$ cut off by x=0, y=0 &
$x^2 + y^2 = 16$ **[2016]**

Q10. Find volume of solid above the xy plane & directly below the position of elliptic paraboloid
$x^2 + \frac{y^2}{4} = z$ which is cut off by the plane $z = 9$ **[2017]**

Q11. Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ revolve about the x axis. Find volume of solid of revolution.
[2018]

MATHEMATICS PAPER I: Analytic Geometry

TUTORIAL SHEET 16: (Plane)

Q1. Find the equation of the plane parallel to the plane $ax+by+cz=0$ and passing through the point (α,β,γ).

Q2. Find the equation of a plane- xy plane and passing through the points $(1,0,5),(0,3,1)$.

Q3. Find the equation of the plane passing through the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point $(1,1,1)$.

Q4. Find the equation of the plane through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

Q5. the plane $lx+my=0$ is rotated about its line of intersection with the plane $z=0$ through an angle α . Prove that the equation of plane in its new position is $lx+my\pm z\sqrt{l^2+m^2}\tan\alpha=0$.

Q6. A variable plane is at a constant distance p from the origin and meets the axes, which are rectangular in A,B,C. Prove that the locus of the point of intersection of the planes through A,B,C parallel to coordinate planes is

$$x^{-2}+y^{-2}+z^{-2}=p^{-2}$$

Q7. Prove that $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ represents a pair of planes.

Q8. From a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the coordinate axes is A',B',C'. Prove that the area of the ΔABC is $\frac{r^5}{2x'y'z'}$ where $r=0P$

Q9. Two system of rectangular axes have the same origin. A plane cuts off intercepts a,b,c,a',b',c' from the axes respectively. Prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$.

Q10. A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show that the locus of the centroid of tetrahedron O,A,B,C is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$.

Q11. A point P moves on a fixed plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. The plane through P perpendicular to OP meets the axis in A,B,C. The plane through A,B,C parallel to coordinate axes intersect in θ. Show that the locus of θ

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$$

Q12. The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$. Find the equation of the plane in its new position.

[2008]

Q13. Find the equation of plane which passes through the point (0, 1, 1) & (2, 0, -1) and is parallel to the line joining (-1, 1, -2) (3, -2, 9). Find also the distance between the line and the plane **[2013]**

Q14. Obtain the equation of plane passing through the point (2, 3, 1) and (4, -5, 3) parallel to x-axis. **[2015]**

Q15. A plane passes through a fixed point (a, b, c) & cuts the axis at point A, B, C respectively. Find the locus of centre of sphere which passes through origin O, A, B, C. **[2017]**

Q16. Find the equation of plane parallel to $3x-y+3z=8$ and passes through point (1, 1, 1)

[2018]

Q17. The plane $x+2y+3z=12$ cuts the axes of coordinate in A, B, C. Find the equation of circle circumscribing the triangle ABC. **[2019]**

TUTORIAL SHEET 17: Straight Line

Q1. A line with direction ratios 2, 7, -5 is drawn to intersect the lines

$$\frac{x}{3}=\frac{y-1}{2}=\frac{z-2}{4} \text{ and } \frac{x-11}{3}=\frac{y-5}{-1}=\frac{z}{1}$$

Find the coordinates of the points of intersection and the length intercepted on it.

[2007]

Q2. Find the locus of the point which moves so that its distance from the plane $x+y-z=1$ is twice its distance from the line $x=-y=z$. **[2007]**

Q3. Find the image of the point $(1,2,3)$ in the plane $2x-3y+6z+35=0$. **[2007]**

Q4. A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ to meet two fixed lines $y=mx, z=c$ and $y=-mx, z=-c$. Find the locus of the line. **[2008]**

Q5. Find the equations of the straight line through the point $(3,1,2)$ to intersect the straight line $x+4=y+1=2(z-2)$ and parallel to the plane $4x+5z+y=0$. **[2011]**

Q6. Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.

Q7. Find the S.D. between the lines and its equation $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

Q8. Find the shortest distance between the lines and its equation $3x-9y+5z=0=x+y-z$ and $6x+8y+3z-13=0=x+2y+z-3$.

Q9. Show that the equation to the plane containing the line $\frac{y}{6}+\frac{z}{c}=1;\ x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1,\ y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if 2d is the S.D. prove that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$.

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Q10. Prove that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0, z+x=0, x+y=0, x+y+z=a$ is $\frac{2a}{\sqrt{6}}$ and three lines of S.D. intersect at the point $x=y=z=-a$.

Q11. If the axes are rectangular the S.D. between the line $y=az+b, z=\alpha x+\beta; y=a'z+b', z=\alpha' x+\beta'$ is

$$\frac{(\alpha-\alpha')(b-b')-(\alpha'\beta-\alpha\beta')(a-a')}{\left[\alpha^2\alpha'^2(a-a')^2+(\alpha-\alpha')^2+(a'\alpha'-a\alpha)^2\right]^{1/2}}$$

Q12. Prove that the lines $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-4}{5}$ and $2x-3y+z=0=x+y-2z+20$ are coplanar. Find also their point of intersection.

Q13. Find the equation of the plane through the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{l}$ and $\frac{x}{n}=\frac{y}{l}=\frac{z}{m}$.

Q14. Prove that all lines which intersect the lines $y=mx, z=c; y=-mx, z=-c;$ and the x-axis lie on the surface $mxz=cy$.

Q15. Prove that locus of a variable line which intersects the three gives lines $y=mx, z=c; y=-mx, z=-c, y=z, mx=-c$ is the surface ___________(Please Check)

Q16. Verify if lines

$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$

& $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$

Are coplanar. If yes, then find the equation of plane in which they lie. **[2015]**

Q17. Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{4}=z-3$ and $y-mx=z=0$. For what values of m will the two lines intersect? **[2016]**

Q18. Find the surface generated by a line which intersects the line $y=a=z$, $x+3z=a=y+z$ and parallel to plane $x+y=0$ **[2016]**

Q19. Find the shortest distance between the skew lines $\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1}$ & $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ **[2017]**

Q20. Find the projection of straight line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1}$ on the plane $x + y + 2z = 6$ **[2018]**

Q21. Find the shortest distance between the lines $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ & z axis. **[2018]**

Q22. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ intersects. Find the coordinates of the point of intersection and the equation of the plane containing them. **[2019]**

TUTORIAL SHEET 18: SPHERE

Q1. Find the equation of sphere passing through the points $(0,0,0)(0,1,,-1)(-1,2,0)(1,2,3)$, coordinates of centre and its radius.

Q2. Find the equation to the sphere through the circle $x^2+y^2+z^2=9,\ 2x+3y+4z=5$ and the origin.

Q3. Find the equation of Tangent planes to the sphere $x^2+y^2+z^2-zx+4y-6z+13=0$ which are parallel to the plane $x-y+z=0$

Q4. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touch the plane $4x+3y=15$

Q5. A sphere of radius k passes through the origin and meets the axes at A,B,C. Prove that the centroid of the triangle ABC lies on the sphere $a\left(x^2+y^2+z^2\right)=4k^2$

Q6. A plane through a fixed point (a,b,c) cuts the axes in A,B,C. Show that the locus of the centre of sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$, O being the origin

Q7. Find out the equation of sphere passing through the origin and meeting the axis of x,y,z respectively at A,B,C.

Q8. Prove that the circle $x^2+y^2+z^2-2x+3y+4z-5=0, 5y+6z+1=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on the same sphere. Also find the value of a for which $x+y+z=\frac{a}{\sqrt{3}}$ touches the sphere.

Q9. Find the equation of a sphere which touches the sphere $x^2+y^2+z^2+2x-6y+1=0$ at $(1,2,-2)$ and passes through the origin.

Q10. If any tangent plane to the sphere $x^2+y^2+z^2=r^2$ makes intercepts a,b,c on the coordinate axes, prove that $(a^{-2}+b^{-2}+c^{-2})=r^{-2}$

Q11. Show that the spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0$ intersect orthogonally. Find their planes of intersection.

Q12. Prove that the centres of spheres which touch the lines $y=mx \quad z=c; \quad y=-mx, \quad z=-c$ lie upon the conicoid $mxy+cz\left(1+m^2\right)=0$

[1993]

Q13. A sphere of constant radius r passes through the origin O and cuts the axes in A,B,C. Prove that the focus of the foot of the perpendicular from O to the line ABC is given by $(x^2+y^2+z^2)(x^{-2}+y^{-2}+z^{-2})=4r^2$ **[2003]**

Q14. Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at the point $\left(1,-2,1\right)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$ **[2006]**

Q15. Show that the sphere $x^2+y^2+z^2-x+z-2=0$ & $3x^2+3y^2-8x-10y+8z+14=0$ cut orthogonally. Find the centre and radius of their common circle. **[2007]**

Q16. Find the equation of a sphere inscribed in the tetrahedron whose faces are $x=0, y=0, z=0, 2x+3y+6z=6$ **[2007]**

Q17. Find the equation (in symmetric form) of the tangent line to the sphere $x^2+y^2+z^2+5x-7y+2z-8=0$, $3x-2y+4z+3=0$ at the point (-3, 5, 4) **[2008]**

Q18. Find the equation of sphere having its centre on the plane $4x-5y-z=3$ & passing through the circle $x^2+y^2+z^2-12x-3y+4z+8=0$, $3x+4y-5z+3=0$ **[2009]**

Q19. Show every sphere through the circle $x^2+y^2-2ax+r^2=0$, $z=0$ cut orthogonally every sphere through the circle $x^2+z^2=r^2$, $y=0$ **[2010]**

Q20. Show that the plane $x+y-2z=3$ cut the sphere $x^2+y^2+z^2-x+y=2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle. **[2010]**

Q21. Show that the equation of the sphere which touches the sphere $4(x^2+y^2+z^2)+10x-25y-2z=0$ at the point (1, 2, -2) and passes through the point (-1, 0, 0) is $x^2+y^2+z^2+2x-6y+1=0$ **[2011]**

Q22. A variable plane is parallel to the given plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes in A, B, c

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+xz\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$$ **[2012]**

Q23. A sphere S has points (0, 1, 0), (3, -5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x-2y+4z+7=0$ as a great circle **[2013]**

Q24. Show that three mutually perpendicular tangent lines can be drawn to sphere $x^2+y^2+z^2=r^2$ from any point on the sphere $x^2+y^2+z^2=3r^2$ **[2013]**

Q25. Find the coordinate of the point of sphere $x^2+y^2+z^2-4x+2y=4$, the tangent planes at which are parallel to the plane $2x-y+2z=8$ **[2014]**

Q26. Find the positive value of 'a' for which the plane $ax-2y+2z+12=0$ touch the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ & hence find the point of contact. **[2015]**

Q27. Find the equation of sphere which passes through the circle $x^2+y^2=4$, $z=0$ and is cut by plane $x+2y+2z=0$ in circle of radius 3 **[2016]**

Q28. A plane passes through a fixed point (a, b, c) and cut the axes at the point A, B, C respectively. Find the locus of the centre of sphere which passes through origin O, and A, B, C. **[2017]**

Q29. Show that plane $2x-2y+z+12=0$ touch the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$. Find the point of contact. **[2017]**

Q30. Find the equation of the sphere in xyz plane passing through the points (0, 0, 0) , (0, 1, -1), (-1, 2, 0) and (1, 2, 3). **[2018]**

Q31. Prove that the plane $z=0$ cuts the enveloping cone of the sphere $x^2+y^2+z^2=11$ which has the vertex at (2, 4, 1) in a rectangular hyperbola. **[2019]**

Unit 19: CONE & CYLINDER

Q1. Find the equation of right circular cylinder whose axis is $x = 2y = -z$ and radius 4. Prove that area of cross-section of this cylinder by the plane $z = 0$ is 24π

Q2. Find the equation of the right circular cylinder which passes through circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$

Q3. Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and which envelops the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Q4. Find the equation of the cylinder whose generator are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and passes through the curve $x^2 + y^2 = 16, z = 0$

Q5. Find the equation of the cylinder which intersects the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$ and whose generators are parallel to z - axis.

Q6. Show that the equation to the right circular cylinder described on the circle through three points $(1,0,0)(0,1,0)$ and $(0,0,1)$ as girding curve is $x^2 + y^2 + z^2 - yz - zx - xy = 1$

Q7. Show that $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{\omega^2}{c} = d$

Q8. Find the equation of a cone whose vertex is the point (α, β, γ) and whos generating lines pass through the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$

Q9. Find out the vertex of the cone $2y^2 - 8yz - 4xz - 8xy + 6x - 4y - 2z + 5 = 0$

Q10. Find the equation of the cone whose vertex is $(1,2,3)$ and griding curve is the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$

Q11. Find the equation of the cone with vertex at $(0,0,0)$ and which passes through the curve $ax^2 + by^2 + cz^2 - 1 = 0 = \alpha x^2 + \beta y^2 - 2z$

Q12. The section of a cone whose vertex is p and guiding curve the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane $x=0$ is a rectangular Hyperbola. Show that focus of p is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$

Q13. Find the equations to the lines in which the plane $2x+y-z=0$ cuts the cone $4x^2-y^2+3z^2=0$

Q14. Prove that the plane $ax+by+cz=0$ cuts the cone $xy+yz+xz=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

Q15. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represent one of a set of three mutually perpendicular generators of the cone $5yz-8xz-3xy=0$ find the equation of the other two

Q16. Prove that cones $ax^2+by^2+cz^2=0$ and $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=0$ are reciprocal

Q17. Prove that the angle between the lines given by $x+y+z=0, ayz+bxz+cxy=0$ is $\frac{\pi}{2}$ if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

Q18. Prove that the equation $\sqrt{fx}+\sqrt{gy}+\sqrt{hz}=0$ represent a cone which touches the coordinate planes and that the equation of its reciprocal cone is $fyz+gzx+hxy=0$

Q19. Find the laws of the vertices of enveloping cones by the plane $z=0$ are circles.

Q20. Show that the cone $yz+zx+xy=0$ cut the sphere $x^2+y^2+z^2=a^2$ in two equal circle & find their area. **[2011]**

Q21. A cone has for its guiding curve the circle $x^2+y^2+2ax+2by=0$, $z=0$ & passes through a fixed point (0, 0, c). If the section of the cone by the plane $y=0$ is a rectangular hyperbola, prove vertex lies on fixed circle
$x^2+y^2+z^2+2ax+2by=0$
$2ax+2by+cz=0$ **[2013]**

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Q22. Examine whether the plane $x + y + z = 0$ cut the cone $yz + zx + xy = 0$ in perpendicular line. **[2014]**

Q23. Show that cone 3yz-2zx-2xy=0 has an infinite set of three mutually perpendicular generator. If $\frac{x}{1} = \frac{y}{1} = \frac{z}{2}$ is a generator belonging to one set, find the other two set. **[2016]**

Q24. Find the equation of cone with (0, 0, 1) as the vertex and $2x^2 - y^2 = 4$, $z = 0$ as guiding curve. **[2018]**

TUTORIAL SHEET 20: Conicoids

Q1. Prove that the lines of intersection of pairs of tangent planes to $ax^2+by^2+cz^2=0$ which touch along perpendicular generators lie on the cone

$$a^2(b+c)x^2+b^2(c+a)y^2+c^2(a+b)z^2=0.$$ **[2004]**

Q2. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ to them through the origin generate the cone

$$(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$$ **[2004]**

Q3. Obtain the equation of right circular cylinder on the circle through the points $(a,0,0)(a,b,0)$ and $(0,0,c)$ as the grinding curve. **[2005]**

Q4. Show that the plane $2x-y+2z=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines. **[2007]**

Q5. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represent one of a set of three immutably + generators of the cone $5yz-8xz-3xy=0$ find the equation of other two. **[2008]**

Q6. Prove that the normals from the point (α,β,γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lies on the cone $\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$. **[2009]**

Q7. Find the vertices of the show quadrilateral formed by the four generator of the Hyperboloid and $(14,2,-2)\frac{x^2}{4}+y^2-z^2=49$ passing through $(10,5,1)$. **[2010]**

Q8. Three points P,Q,R are taken on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ so that the lines joining P,Q,R to the origin are mutually perpendicular. Prove that the plane P,Q,R touches a fixed sphere. **[2011]**

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Q9. Show that the generators through any one of the ends of an equicojugate diameter of the principal elliptic section of the Hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of 60^0 if $a^2+b^2=6c^2$. Find also the condition for the generators to be perpendicular to each other. **[2011]**

Q10. Show that the locus of a point from which the these mutually perpendicular tangent lines can be drawn to paraboloid $x^2+y^2+2z=0x^2+y^2+4z=1$. **[2012]**

Q11. A variable generator meets two generators of system through the extremities B and B' of minor axis of principal elliptic section of hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2c^2=1$ are P & P'. Prove that

BP. B$P'=a^2+c^2$ **[2013]**

Q12. Show that the lines drawn from the origin parallel to the normal to the central conicoid $ax^2+by^2+cz^2=1$ at its point of intersection with the plane $lx+my+nz=p$ generates the cone

$p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$ **[2014]**

Q13. Find the equation of two generating lines through any point $(a\,cos\theta\,, b\,sin\theta\,, 0)$ of the principle elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \quad z=0$ of the hyperboloid by the plane $z=0$ **[2014]**

Q14. Two perpendicular tangent plane to paraboloid $x^2+y^2=2z$ intersects in a straight line in the plane x=0. Obtain the curve to which this straight line touches. **[2015]**

Q15. Find the locus of point of intersecting of three mutually perpendicular tangent planes to the conicoid $ax^2+\ by^2+cz^2=1$ **[2016]**

Q16. Reduce the following equation to the standard form & hence determine the value of the conicoid $x^2+y^2+z^2-yz-zx-xy-3x-6y-9z+21=0$ **[2017]**

Q17. Find the equation of generating lines of paraboloid $(x+y+z)(2x+y-z)=6z$ which passes through point (1, 1, 1) **[2018]**

Q18. Prove that in general three normal can be drawn from a given point to the paraboloid $x^2+y^2=2az$ but if the point lies on the surface 27a$(x^2+y^2)+8(a-z)^3=0$ then two of the three normal coincide **[2019]**

Q19. Find the length of the normal chord through a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and prove that if it is equal to 4 PG_3 , where G_3 is the point where the normal chord through P meets the xy plane , then P lies on the cone
$\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$ **[2019]**

TUTORIAL SHEET 21: 2nd Degree Equation, Reduction to Canonical Form

Q1. Reduce the equation:

$11x^2+10y^2+6z^2-8yz+4zx-12xy+7zx-7zy+36z+150=0$ to the standard form and give the nature of the surface. Also find the equations of its axes.

Q2. Prove that the equation $x^2+y^2+z^2+yz+zx+xy+3x+y+4z+4=0$ represents an ellipsoid the squares of whose semi axes are $2,2,\frac{1}{2}$. Show that its principal axis is given by $x+1=y-1=z+2$.

Q3. Show that the equation $2y^2+4zx+2x-4y+6z+5=0$ represents a right circular cone. Show also that the semi-vertical angle of this cone is $\frac{\pi}{4}$ and that its axis is given by $x+z+2=0,\ y=1$.

Q4. Show that $2x^2+2y^2+z^2+2yz-2xz-4xy+x+y=0$

Q5. Reduce $3z^2-6yz-6zx-7x-5y-6z+3=0$ to standard for and find the nature of the surface represented by this equation.

Q6. Prove that $5x^2+5y^2+8z^2+8yz+8zx-2xy+12x-12y+6=0$ represents a cylinder whose cross-section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$ and find the equations to its axis.

Q7. Reduce the surface $36x^2+4y^2+z^2-4yz-12zx+24xy+4x+16y-26z-3=0$ to the standard form and find the lotus rectum of a normal section.

Most general equation of 2nd degree in 3 coordinates:

$F(x,y,z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy+24x+2vy+2wz+d=0$

It can be reduced to any one of the below mentioned forms by transformation of axes:

$\lambda_1x^2+\lambda_2y^2+\lambda_3z^2=\mu$ (1)

$\lambda_1x^2+\lambda_2y^2=2\mu z$ (2)

By giving different values to $\lambda_1,\lambda_2,\lambda_3$ & μ from (1) can be reduced to

(i) $Ax^2+By^2+Cz^2=1$	Ellipsoid
(ii) $A(x^2+y^2)+Cz^2=1$	Ellipsoid of revolution
(iii) $A(x^2+y^2+z^2)=1$	Sphere
(iv) $Ax^2+By^2-Cz^2=1$	Hyperboloid of one sheet
(v) $Ax^2-By^2-Cz^2=1$	Hyperboloid of two sheet
(vi) $A(x^2-z^2)+By^2=1$	Hyperboloid of revolution
(vii) $Ax^2+By^2+Cz^2=0$	Cone
(viii) $Ax^2+By^2=1$	Elliptic cylinder
(ix) $Ax^2-By^2=1$	Hyperbolic cylinder
(x) $Ax^2-By^2=0$	Pair of intersecting planes
(xi) $Ax^2=1$ or $By^2=1$ or $Cz^2=1$	Pair of parallel planes

Homogenous part $f(x,y,z)=ax^2+by^2+(z^2+2fyz+2gzx+2hxy)$.

CIVIL SERVICES EXAMINATION (MAINS)

MATHEMATICS PAPER I: O.D.E

TUTORIAL SHEET 22: Differential equations of First order and First Degree

Q1. Solve $x\frac{dy}{dx} + y\log y = xye^x$ **[2003]**

Q2. Solve $\frac{dy}{dx} + y\cos x = \frac{1}{2}\sin 2x$ **[2004]**

Q3. Solve $y\left(xy + 2x^2y^2\right)dx + x\left(xy - x^2y^2\right)dy = 0$ **[2004]**

Q4. Solve $xy\frac{dy}{dx} = \sqrt{x^2 - y^2 - x^2y^2 + 1}$ **[2005]**

Q5. Solve the D.E $\left(xy^2 + e^{-\frac{1}{x^3}}\right)dx - x^2y\,dy = 0$ **[2006]**

Q6. Solve $\left(1 + y^2\right) + \left(x - e^{-\tan^{-1}y}\right)\frac{dy}{dx} = 0$ **[2006]**

Q7. Solve the D.E

$\cos 3x\frac{dy}{dx} - 3y\sin 3x = \frac{1}{6}\sin 6x + \sin^2 3x \qquad 0 < x < \frac{\pi}{2}$ **[2007]**

Q8. Solve $\frac{dy}{y} + xy^2dx = -4xdx$ **[2007]**

Q9. Solve $ydx + \left(x + x^3y^2\right)dy = 0$ **[2008]**

Q10. Solve $\frac{dy}{dx} = \frac{y^2\left(x - y\right)}{3xy^2 - x^2y - 4y^3}, \qquad y\left(0\right) = 1$ **[2009]**

Q11. Find the D.E. of the family of circles in the xy - plane passing through $\left(-1,1\right)$ and $\left(1,1\right)$.

[2009]

Q12. Show that D.E.

$\left(3y^2 - x\right) + 2y\left(y^2 - 3\right)y' = 0$ admits an integrating factor which is a function of $\left(x + y^2\right)$.

Hence solve the equation. **[2010]**

Q13. Verify that

$$\frac{1}{2}(Mx+Ny)\,d(\ln xy)+\frac{1}{2}(Mx-Ny)\,d\ln\left(\frac{x}{y}\right)=Mdx+Ndy$$

Hence show that:

(i) If the D.E. $Mdx+Ndy=0$ is homogeneous, then $Mx+Ny$ is an I.F. unless $Mx+Ny=0$

(ii) If the D.E $Mdx+Ndy=0$ is not exact but is of the form

$f_1(x,y)\,ydx+f_2(x,y)\,xdy=0$, then $\frac{1}{Mx-Ny}$ is an I.F. unless $Mx-Ny=0$. **[2010]**

Q14. Solve $\frac{dy}{dx}=(4x+y+1)^2$

Q15. Solve $\frac{dy}{dx}=\frac{2xy\,e^{\left(\frac{x}{y}\right)^2}}{y^2\left(1+e^{\left(\frac{x}{y}\right)^2}\right)+2x^2e^{\left(\frac{x}{y}\right)^2}}$ **[2012]**

Q16. Show that the D.E.

$(2xy\log y)\,dx+\left(x^2+y^2\sqrt{y^2+1}\right)dy=0$ is not exact. Find an integrating factor and hence the solution of the equation. **[2012]**

Q17. Solve: $\frac{dy}{dx}=\cos(x+y)+\sin(x+y)$ **[2013]**

Q18. Solve $(5x^3+12x^2+6x)\,dx+6xy\,dy=0$ **[2013]**

Q19. Justify that differential equation of the form
$[y+x\,f(x^2+y^2)]dx+[yf(x^2+y^2)-x]dy=0$
Where $f(x^2+y^2)$ is an arbitrary function of x^2+y^2 is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for
$f(x^2+y^2)=(x^2+y^2)^2$ **[2014]**

Q20. Find the curve for which the part of the tangent cut off by the axis is bisected at the point of tangency. **[2014]**

Q21. Solve $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1.$ **[2015]**

Q22. Solve D.E
$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$ **[2015]**

Q23. Find the constant a so that $(x+y)^a$ is the integrating factor of

$(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve differential equation. **[2015]**

Q24. Find the D.E. representing all the circles in xy plane. **[2016]**

Q25. $\frac{dy}{dx} = \frac{1}{1+x^2}(e^{\tan^{-1}x} - y).$ **[2016]**

Q26. Solve:
$(y(1 - x\tan x) + x^2\cos x)dx - xdy = 0$ **[2016]**

Q27. Find α and β such that $x^{\alpha}y^{\beta}$ is an integrating factor of

$(4y^2 + 3xy)dx - (3xy + 2x^2)dy = 0$ and solve the equation.

Q.28. Solve the following simultaneous linear differential equations:
$(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where y and z are functions of independent variable x and $\frac{d}{dx} = D$ **[2017]**

Q29. Find f(y) such that $(2xe^y + 3y^2)dy + \left(3x^2 + f(y)\right)dx = 0$ is exact and hence solve.
[2018]

Q30. Solve the differential equation
$(2y\sin x + 3y^4\sin x\cos x)dx - (4y^3\cos^2 x + \cos x)dy = 0$ **[2019]**

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TUTORIAL SHEET 23: D.E. of 1st order and Higher Degree

Q1. Solve the D.E $\left(px^2+y^2\right)\left(px+y\right)=\left(p+1\right)^2$ by reducing to Clairaut's form using suitable substitutions. **[2003]**

Q2. Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal **[2003]**

Q3. Reduce the equation to Clairaut's equation and solve it:

$$\left(px-y\right)\left(py+x\right)=2p \text{ where } p=\frac{dy}{dx}$$ **[2004]**

Q4. Solve the D.E by reducing to it to Clairaut's from by using suitable substitution

$(x^2+y^2)(1+p)^2-2(x+y)(1+p)(x+yp)+(x+yp)^2=0$ **[2005]**

Q5. Find the orthogonal trajectory of a system of coaxial circles $x^2+y^2+2gx+c=0$ where g is the parameter. **[2005]**

Q6. Solve $x^2p^2+yp\left(2x+y\right)+y^2=0$, using the substitution $y=u$ and $xy=v$ and find its singular solution where $p=\frac{dy}{dx}$ **[2006]**

Q7. Find the family of curves whose tangents form an angle $\frac{\pi}{4}$ with the hyperbolas $xy=c, c>0$. **[2006]**

Q8. Determine the general and singular solution of the equation

$$y=x\frac{dy}{dx}+a\frac{dy}{dx}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-\frac{1}{2}}$$

, a being a constant **[2007]**

Q9. Solve the equation $y-2xp+yp^2=0$ where $p=\frac{dy}{dx}$. **[2008]**

Q10. Determine the orthogonal trajectory of a family of curves represented by the polar equation $r=a\left(1-\cos\theta\right)$. **[2011]**

Q11. Obtain Clairut's form of the D.E.

$\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+x\right)=a^2\frac{dy}{dx}$. Also find its general solution. **[2011]**

Q12. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = ax$. **[2012]**

Q13. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a\sin n\theta$ **[2013]**

Q14. Solve D.E.

$x = py - p^2$, where $p = \frac{dy}{dx}$ **[2015]**

Q15. Show the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. **[2016]**

Q16. Consider D.E. $xyp^2 - (x^2+y^2-1)p + xy = 0$, where $p = \frac{dy}{dx}$ substituting $u = x^2$ and $v = y^2$ reduce the equation to clairut's form in terms of u, v and $p' = \frac{dv}{du}$. Hence or otherwise solve the equation. **[2017]**

Q17. Solve

$$\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$$ **[2018]**

Q18. Obtain the singular solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 \left(\frac{y}{x}\right)^2 cot^2\alpha - 2\frac{dy}{dx}\frac{y}{x} + \left(\frac{y}{x}\right)^2 cosec^2\alpha = 1$$

Also find the complete primitive of the given differential equation. Give the geometrical interpretation of the complete primitive and singular solution. **[2019]**

TUTORIAL SHEET 24: D.E. of 2nd order with constant coefficients

Q1. Solve $(D^5 - D)y = 4(e^x + \cos x + x^3)$ where $D = \frac{d}{dx}$. **[2003]**

Q2. Solve $(D^4 - 4D^2 - 5)y = e^x(x + \cos x)$ **[2004]**

Q3. Solve $(D^2 - 2D + 2)y = e^x \tan x$ by method of variation of parameters **[2006]**

Q4. Solve $(D^3 - 6D^2 + 12D - 8)y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right)$ **[2007]**

Q5. Obtain the general solution: $y'' - 2y' + 2y = x + e^x \cos x$ **[2011]**

Q6. Find the general solution of the equation $y''' - y'' = 12x^2 + 6x$ **[2012]**

Q7. By Method of variation of Parameters, solve the D.E.

$(D^2 + 2D + 1)y = e^{-x}\log(x)$ **[2016]**

Q8. Find particular integral of

$\frac{d^2y}{dx^2} + y = e^{\frac{x}{2}} \sin \frac{x\sqrt{3}}{2}$ **[2016]**

Q9. Solve D.E. using method of variation of parameters:

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 44 - 76x - 48x^2$ **[2017]**

Q10. Solve initial value D.E. **[2017]**

$20y'' + 4y' + y = 0$

y(0)= 3.2 y'(0)=0

Q11. Solve $y'' - y' = x^2 e^{2x}$. **[2018]**

Q12. Solve $y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$.

[2018]

Q13. Solve $y'' + 16y = 32\sec 2x$.

Q14. Solve initial value problems **[2018]**

$y'' - 5y' + 4y = e^{2t}$

y(0)=$\frac{19}{12}$ y'(0)=$\frac{8}{3}$

Q15. Determine the complete solution of differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x$

[2019]

'TUTORIAL SHEET 25: 2nd Order D.E. with Variable Coefficients

Q1. Solve $(1+x)^2 y'' + (1+x) y' + y = \sin\{2\log(1+x)\}$. **[2003]**

Q2. Solve the D.E. by Variation of parameters: $x^2 y'' - 4xy' + 6y = x^4 \sec^2 x$ **[2003]**

Q3. Solve $(x+2)\dfrac{d^2y}{dx^2} - (2x+5)\dfrac{dy}{dx} + 2y = (x+1)e^x$ **[2004]**

Q4. Solve: $(1-x^2)\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} - (1+x^2)y = x$ **[2004]**

Q5. Solve: $\left[(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)\right] y = \dfrac{1}{x+1}$. **[2005]**

Q6. Solve the D.E.: $(\sin x - x\cos x) y'' - x\sin x y' + y\sin x = 0$, given that $y = \sin x$ is a solution of this equation. **[2005]**

Q7. Solve the D.E. by variation of parameters: $x^2 y'' - 2xy' + 2y = x\log x, \quad x > 0$ **[2005]**

Q8. Solve: $x^2 \dfrac{d^3y}{dx^3} + 2x\dfrac{d^2y}{dx^2} + 2\dfrac{y}{x} = 10\left(1 + \dfrac{1}{x^2}\right)$ **[2006]**

Q9. Solve: $2x^2 \dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - 3y = x^3$ **[2007]**

Q10. Solve by the method of variation of parameters: $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = 2e^x$ **[2007]**

Q11. Use the method of variation of parameters to find the general solution of $x^2 y'' - 4xy' + 6y = -x^4 \sin x$. **[2008]**

Q12. Solve the D.E.: $x^3 y'' - 3x^2 y' + xy = \sin(\ln x) + 1$. **[2008]**

Q13. Solve by the method of Variation of parameters: $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$. **[2011]**

Q14. Solve the D.E.: $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$ **[2012]**

Q15. Using the method of variation of parameters, solve $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$. **[2013]**

Q16. Find the general solution of $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \ln x \sin(\ln x)$ **[2013]**

Q17. Solve the D.E.: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\ln x)$ **[2014]**

Q18. Solve the following D.E.: $x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x}$, when e^x is a solution to its corresponding homogenous D.E. **[2014]**

Q19. Solve by the method of variations of parameters $\frac{dy}{dx}-5y=\sin x$. **[2014]**

Q18. Solve the following D.E. :

$$x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x}$$

,when is a solution to its corresponding homogenous D.E. **[2014]**

Q19. Solve:

$$x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2\cos(\log_e x)$$

[2015]

Q20. Find the general solution of the equation

$x^2\frac{d^3y}{dx^3}-4x\frac{d^2y}{dx^2}+6\frac{dy}{dx}=4$ **[2016]**

Q21. Solve the D.E.

$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2)$ **[2017]**

Q22. Solve

$(1+x)^2y''+(1+x)y'=4\cos(\log(1+x))$ **[2018]**

Q23. Solve $\frac{d^2y}{dx^2}+(3\sin x-\cot x)\frac{dy}{dx}+2y\, sin^2 x=e^{-cosx}sin^2x$ **[2019]**

Q24. Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y''-2xy'+2y=x^3\sin x$ and then find the general solution of the given equation by the method of variation of parameters. **[2019]**

TUTORIAL SHEET 26: Laplace Transforms

Q1. Using Laplace transform, solve the initial value problem $y''-3y'+2y=4t+e^{3t}$ with $y(0)=1, \quad y'(0)=-1$ **[2008]**

Q2. Find the inverse Laplace transform of $F(s)=\ln\left(\frac{s+1}{s+5}\right)$. **[2009]**

Q3. Use Laplace transform to solve: $\frac{d^2y}{dx^2}-2\frac{dx}{dt}+x=e^t, \quad x(0)=2$ and $\left.\frac{dx}{dt}\right|_{t=0}=-1$.

[2011]

Q4. Using Laplace transforms, solve the initial value problem

$$y''+2y'+y=e^{-t}, \quad y(0)=-1, y'(0)=1$$ **[2012]**

Q5. Using Laplace transform method, solve: $(D^2+n^2)x=a\sin(nt+\alpha)$ with condition at $x=0$ and $\frac{dx}{dt}=0$ at $t=0$. **[2013]**

Q6. Solve the initial value problem using Laplace transform:

$$\frac{d^2y}{dt^2}+y=8e^{-2t}\sin t, \quad y(0)=0, y'(0)=0.$$ **[2014]**

Q7. Obtain Laplace Inverse transform of

(i) $\left\{\left(1+\frac{1}{s^2}\right)+\left(\frac{s}{s^2+2s}\right)(e^{-\pi s})\right\}$

(ii) Using Laplace transform solve

$y''+y=t, \; y(0)=1, y'(0)=2$ **[2015]**

Q8. Using Laplace transformation, solve following

$y''-2y'-8y=0, y(0)=3, \; y'(0)=6$ **[2016]**

Q9. Solve initial value problems using Laplace transform

$$\frac{d^2y}{dx^2}+9y=r(x), y(0)=0, \; y'(0)=4$$

Where, r(x) = $\begin{cases} 8\sin x & if\ 0<x<\pi \\ 0 & if\ x\geq\pi \end{cases}$ **[2017]**

Q10. (i) Find the Laplace transform of f(t)= $\frac{1}{\sqrt{t}}$. **[2018]**

(ii) $\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$ Find the inverse Laplace transform.(10marks)

Q11. Find the Laplace transform of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where n∈ N, is $\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$ **[2019]**

Mathematical Paper 1: Section B / Vector Analysis

TUTORIAL SHEET 27: Scalar & Vector Fields, Triple Products, Differentiation of vector

Q1. If $\vec{a}\times\vec{r}=\vec{b}+\lambda\vec{a}$ and $\vec{a}.\vec{r}=3$ where $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{j}+\hat{k}$, then find $\vec{r}$ and λ

Q2. If $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\vec{b}}{2}$. Find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, $\vec{b}$ and $\vec{c}$ being non parallel.

Q3. If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the vertices A,B,C of a triangle, show that vector area of the Δ is $\frac{1}{2}.\left(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}\right)$

Q4. Prove that $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left(a\,b\,c\right)$

Q5. Show that the four points whose position vectors are $3\hat{i}-2\hat{j}+4\hat{k}, 6\hat{i}+3\hat{j}+\hat{k}, 5\hat{i}+7\hat{j}+3\hat{k}$ and $2\hat{i}+2\hat{j}+6\hat{k}$ are coplanar

Q6. Prove the identity

(i) $\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{c}\times\vec{d}\right)=\begin{vmatrix}\vec{a}.\vec{c} & \vec{a}.\vec{d}\\ \vec{b}.\vec{c} & \vec{b}.\vec{d}\end{vmatrix}$

(ii) $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=\left[\vec{a}\vec{b}\vec{d}\right]\vec{c}-\left[a\vec{b}\vec{c}\right]\vec{a}$

Q7. If $\frac{d\vec{u}}{dt}=\vec{\omega}\times\vec{u},\frac{d\vec{v}}{dt}=\vec{\omega}\times\vec{v}$, show that $\frac{d}{dt}\left(\vec{u}\times\vec{v}\right)=\vec{\omega}\times\left(\vec{u}\times\vec{v}\right)$

Q8. If $\vec{R}$ be a unit vector in the direction of $\vec{r}$, prove that $R\times\frac{d\vec{R}}{dt}=\frac{1}{r^2}\vec{r}\times\frac{d\vec{r}}{dt}$

Q9. If $\vec{r}\left(t\right)=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$, prove that $\int_1^2\left(\vec{r}\times\frac{d^2r}{dt^2}\right)dt=-14\hat{i}+75\hat{j}-15\hat{k}$

Q10. Let $\vec{R}$ be the unit vector along the vector $\vec{r}\left(t\right)$. Show that $\vec{R}\times\frac{d\vec{R}}{dt}=\frac{\vec{r}}{r^2}\times\frac{d\vec{r}}{dt}$ **[2002]**

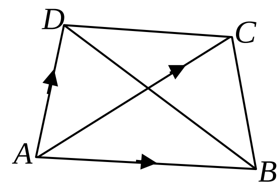
Q11. Show that if $\vec{a}', \vec{b}' \& \vec{c}'$ are the reciprocals to the non coplanar vector $\vec{a}, \vec{b}, \vec{c}$, then any vector $\vec{r}$ may be written as

$$\vec{r} = (\vec{r}\cdot\vec{a}')\vec{a} + (\vec{r}\cdot\vec{b}')\vec{b} + (\vec{r}\cdot\vec{c}')\vec{c}$$ **[2003]**

Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$ where t is the time.

Q12. Find the components of its acceleration along the radial and transverse directions. **[2003]**

Q13. Show that the volume of tetrahedron ABCD is $\frac{1}{6}(\vec{AB}\times\vec{AC})\cdot\vec{AD}$. Hence find the volume of the tetrahedron with vertices $(2,2,2),(2,0,0),(0,2,0)\&(0,0,2)$.



[2005]

Q14. If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} - 7\hat{k}$ determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B} = \vec{C}\times\vec{B}$ and $\vec{R}.\vec{A} = 0$ **[2006]**

Q15. Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential $\vec{F}$ and work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.

[2008]

Q16. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ z = 0$ under the field of force given by

$$\vec{F} = (2x - y + z)\hat{i} + (x + y - z)\hat{j} + (3x - 2y + 4z)\hat{k}.$$ **[2009]**

Q17. Show that the vector field defined by the vector function $\vec{V} = xyz(yz\hat{i} + xz\hat{j} + xy\hat{k})$ is conservative. **[2010]**

Q18. For any vectors $\vec{a}\&\vec{b}$ given respectively by $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin t\hat{i} - \cos t\hat{j}$ determine (i) $\frac{d}{dt}(\vec{a}.\vec{b})$ and (ii) $\frac{d}{dt}(\vec{a}\times\vec{b})$. **[2011]**

Q19. Examine within the vectors $\vec{\nabla}u, \vec{\nabla}v \,\&\, \vec{\nabla}w$ are coplanar, where u, v, w are the scalar function whether defined by

$$u = x + y + z$$

$$v = x^2 + y^2 + z^2$$

$$w = yz + xz + xy$$

[2011]

Q20. If $\vec{A} = x^2yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$

$\vec{B} = 2z\hat{i} - y\hat{j} - x^2\hat{k}$

Find the value of $\frac{\partial^2}{\partial xy}\left(\vec{A} \times \vec{B}\right)$ at $(1, 0, -2)$. **[2012]**

Q21. A vector field is given by

$\vec{F} = (x^2 + xy^2)\hat{\imath} + (y^2 + x^2y)\hat{\jmath}$. Verify that the field $\vec{F}$ is irrotational or not. Find scalar potential **[2015]**

Q22. Prove that the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\vec{b} = -\hat{\imath} + 3\hat{\jmath} + 4k$, $\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$ can form the side of a triangle. Find the lengths of the medians of the triangle. **[2016]**

Q23. The position vector of a moving point a time t is $\vec{r} = \sin t\ \hat{\imath} + \cos 2t\,\hat{\jmath} + (t^2 + 2t)\hat{k}$. Find the component of acceleration $\vec{a}$ in the directions parallel to velocity $\vec{v}$ and perpendicular to the plane of $\vec{r}$ and $\vec{v}$ at time $t = 0$ **[2017]**

CIVIL SERVICES EXAMINATION (MAINS)

TUTORIAL SHEET 28: Gradient, Divergence and Curl

Q1. If $\vec{a}\,\&\,\vec{b}$ are constant vectors, then show that:

(i) $\vec{\nabla}\cdot\left\{\vec{x}\times\left(\vec{a}\times\vec{x}\right)\right\}=-2\vec{x}\cdot\vec{a}$

(ii) $\vec{\nabla}\cdot\left\{\left(\vec{a}\times\vec{x}\right)\times\left(\vec{b}\times\vec{x}\right)\right\}=2\vec{a}\cdot\left(\vec{b}\times\vec{x}\right)-2\vec{b}\cdot\left(\vec{a}\times\vec{x}\right)$ **[1992]**

Q2. Prove that the angular velocity of rotation at any point is equal to one half of the curl of the velocity vector $\vec{V}$. **[1993]**

Q3. Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n=-3$. **[1994, 2006]**

Q4. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=\left|\vec{r}\right|$, show that:

(i) $\vec{r}\times\text{grad}\,f\left(r\right)=0$

(ii) $\vec{\nabla}\cdot\left(r^n\vec{r}\right)=\left(n+3\right)r^n$ **[1996]**

Q5. If $\vec{r}_1$ and $\vec{r}_2$ are the vectors joining the fixed points $A\left(x_1,y_1,z_1\right)$ and $B\left(x_2,y_2,z_2\right)$ respectively to a variable point $P\left(x,y,z\right)$, then find the values of Grad $\left(\vec{r}_1\cdot\vec{r}_2\right)$ and $\vec{r}_1\times\vec{r}_2$. **[1998]**

Q6. Evaluate $\vec{\nabla}\times\vec{F}$ for $\vec{F}=\vec{\nabla}\left(x^3+y^3+z^3-3xyz\right)$. **[1999]**

Q7. In what direction from the point $\left(-1,1,1\right)$ is the directional derivative of $f=x^2yz^3$ is maximum? Compute the magnitude. **[2000]**

Q8. Show that the vector field defined by $\vec{F}=2xyz^3\hat{i}+x^2z^3\hat{j}+3x^2yz^2\hat{k}$ is irrotational. Find also the scalar U such that $\vec{F}=\text{Grad}\,U$. **[2001]**

Q9. Find the directional derivative of $f=x^2yz^3$ along $x=e^{-t}$, $y=1+2\sin t$, $z=t-\cos t$ at $t=0$. **[2001]**

Q10. Show that $\vec{\nabla}\times\dfrac{\left(\vec{a}\times\vec{b}\right)}{r^3}=-\dfrac{\left(\vec{a}\times\vec{r}\right)}{r^3}=-\dfrac{\vec{a}}{r^3}+\dfrac{3\vec{r}}{r^5}\left(\vec{a}\cdot\vec{r}\right)$, where $\vec{a}$ is any constant vector.

[2001]

Q11. Show that Curl $\left(\text{curl } \vec{V}\right) = \text{Grad}\left(\text{div}\vec{V}\right) - \nabla^2\vec{V}$ **[2002]**

Q12. Prove that the divergence of a vector field is invariant with respect to coordinate transformations. **[2003]**

Q13. Prove the identity: $\vec{\nabla}\left(A^2\right) = 2\left(\vec{A}\cdot\vec{\nabla}\right)\vec{A} + 2\vec{A}\times\left(\vec{\nabla}\times\vec{A}\right)$, where $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

[2003]

Q14. Prove the identity: $\vec{\nabla}\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} + \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} + \vec{B}\times\left(\vec{\nabla}\times\vec{A}\right) + \vec{A}\times\left(\vec{\nabla}\times\vec{B}\right)$ **[2004]**

Q15. Show that if $\vec{A}$ & $\vec{B}$ are irrotational, then $\vec{A}\times\vec{B}$ is solenoidal. **[2004]**

Q16. Prove that the curl of a vector field is independent of the choice of coordinates. **[2005]**

Q17. Show that $\vec{\nabla}\times\left(\hat{k}\times grad\,\frac{1}{r}\right) + grad\left(\hat{k}\cdot grad\,\frac{1}{r}\right) = 0$, where r is the distance from the origin and $\vec{k}$ is the unit vector in the direction OZ. **[2005]**

Q18. Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x$ at the point $1, 2, -1$ has maximum magnitude 64 in the direction parallel to z - axis. **[2006]**

Q19. Prove that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n + 3 = 0$.

[2006]

Q20. If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}$, $r = |\vec{r}|$ determine $\vec{\nabla}\left(\frac{1}{r}\right)$ in terms of $\hat{r}$ and r. **[2007]**

Q21. For any constant vector $\vec{a}$, show that the vector represented by $\vec{\nabla}\times\left(\vec{a}\times\vec{r}\right)$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z) measured from the origin. **[2007]**

Q22. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find the value(s) of n in order that $r^n\vec{r}$ may be (i) Solenoidal (ii) Irrotational. **[2007, 2011]**

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Q23. Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ where $r = \left(x^2+y^2+z^2\right)^{1/2}$. Hence find $f(r)$ such that $\nabla^2 f(r) = 0$. **[2008]**

Q24. Show that $\vec{\nabla}\cdot\left(\vec{\nabla} r^n\right) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2+y^2+z^2}$. **[2009]**

Q25. Find the directional derivative of

(i) $4xz^3 - 3x^2y^2z^2$ at $(2,-1,2)$ along Z - axis.

(ii) $x^2yz + 4xz^2$ at $(1,-2,1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **[2009]**

Q26. Find the directional derivative of $f(x,y) = x^2y^3 + xy$ at the point $(2,1)$ in the direction of a unit vector which makes an angle of $\frac{\pi}{3}$ with the x - axis. **[2010]**

Q27. Prove that $\vec{\nabla}\cdot\left(f\vec{\nabla}\right) = f\left(\vec{\nabla}\cdot\vec{V}\right) + \left(\vec{\nabla} f\right)\cdot\vec{V}$, where f is a scalar function. **[2010]**

Q28. If u and v are two scalar fields and $\vec{f}$ is the vector field such that $u\vec{f} = \vec{\nabla}(V)$, find the value of $\vec{f}\cdot\left(\vec{\nabla}\times\vec{f}\right)$. **[2011]**

Q29. Calculate $\nabla^2\left(r^n\right)$ and find its expression in terms of vector, r being the distance of any point (x,y,z) from the origin, n being constant and ∇^2 being Laplace operator. **[2013]**

Q30. Find f(r) such that $\Delta f = \frac{\vec{r}}{r^5}$and f(1)=0. **[2016]**

Q31. Find what values of constants a,b and c. The vector $\vec{v} = (x+y+az)\hat{\imath} + (bx+2y-z)\hat{\jmath} + (-x+y+2z)\hat{k}$ is irrational. Find the divergence in cylindrical coordinate of this vector with these values. **[2017]**

Q32. Find the angle between the tangent at a general point of the curve whose equations are $x = 3t,\ y = 3t^2,\ z = 3t^3$ and the line $y = z - x = 0$ **[2018]**

Q33. Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve x=t, y=t^2,$z = t^3$ at the point (1, 1, 1) **[2019]**

CIVIL SERVICES EXAMINATION (MAINS)

TUTORIAL SHEET 29: Curvature and Torsion

Q1. Find the length of the arc of the twisted curve $\vec{r} = \left(3t, 3t^2, 2t^3\right)$ from the point $t = 0$ to the point $t = 1$. Find also the unit tangent t, unit normal n and the unit binormal b at $t = 1$.

[2001]

Q2. Find the curvature k for the space curve:

$$x = a\cos\theta, \quad y = a\sin\theta, \quad z = a\theta\tan\alpha$$ **[2002]**

Q3. Find the radii of curvature and torsion at a point of intersection of the surfaces $x^2 - y^2 = c^2$, $y = x\tanh\frac{x}{c}$. **[2003]**

Q4. Show that the Frenet-Serret Formula can be written in the form

$$\frac{d\vec{T}}{dS} = \vec{\omega}\times\vec{T}, \quad \frac{d\vec{N}}{dS} = \vec{\omega}\times\vec{N}, \quad \frac{d\vec{B}}{dS} = \vec{\omega}\times\vec{B} \text{ where } \vec{\omega} = \tau\vec{T} - k\vec{B}$$ **[2004]**

Q5. Find the curvature and the torsion of the space curve

$$x = a\left(3u - u^3\right), \; y = 3au^2, \; z = a\left(3u + u^2\right).$$ **[2005]**

Q6. The parametric equation of a circular helix is $\vec{r} = a\cos u\hat{i} + a\sin u\hat{j} + cu\hat{k}$, where c is a constant and u is a parameter. Find the unit tangent vector $\hat{t}$ at the point u and the arc length measured from $u = 0$. Also find $\frac{d\hat{t}}{dS}$ where S is the arc length. **[2005]**

Q7. If the unit tangent vector $\vec{t}$ and binomial $\vec{b}$ makes angles θ & ϕ respectively with a constant unit vector $\vec{a}$, prove that $\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = -\frac{k}{\tau}$. **[2006]**

Q8. Find the curvature and torsion at any point of the curve:

$$x = a\cos 2t \qquad y = a\sin 2t \qquad z = 2a\sin t.$$ **[2007]**

Q9. Show that for the space curve

$$x = t, \; y = t^2 \; z = \frac{2}{3}t^3$$

The curvature and torsion are same at every point. **[2008]**

Q10. Find $\frac{k}{\tau}$ for the curve $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$ **[2010]**

Q11. Derive the Frenet-Serret Formulae. Define the curvature and torsion for a space curve. Compute them for the space curve

$$x = t, \quad y = t^2 \quad z = \frac{2}{3}t^3.$$

Show that the curvature and torsion are equal for the curve. **[2012]**

Q12. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\,\hat{j} - t\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. **[2013]**

Q13. Show that the curve $\vec{x}(t) = t\hat{i} + \frac{1+t}{t}\hat{j} + \frac{1-t^2}{t}\hat{k}$ lies in a plane. **[2013]**

Q14. Find the curvature vector at any point of the curve $\vec{r}(t) = t\cos t\hat{i} + t\sin t\hat{j},\ 0 \le t \le 2\pi$. Give its magnitude. **[2014]**

Q15. Find the curvature vector and its magnitude at any point $\vec{r} = (\theta)$ of the curve $\vec{r} = (a\cos\theta, a\sin\theta, a\theta)$. Show that the lower of the feet of perpendicular from the origin to the tangent is a curve that completely lies on hyperboloid $x^2+y^2-z^2=a^2$.
[2017]

Q16. Find the curvature and torsion of the curve

$\vec{r} = a(u - \sin u)\hat{\imath} + a(1 - \cos u)\hat{\jmath} + bu\hat{k}$ **[2018]**

Q17. Find the radius of curvature and radius of torsion of the helix

$x = a\cos u, y = a\sin u, \ z = au\tan\alpha$ **[2019]**

CIVIL SERVICES EXAMINATION (MAINS)

TUTORIAL SHEET 30: Line, Surface and Volume Integrals

Q1. Evaluate $\iint_S \vec{\nabla}\times\vec{F}\cdot\hat{n}\,dS$ where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and $\vec{F}=z\hat{i}+x\hat{j}+y\hat{k}$ **[1993]**

Q2. If $\vec{F}=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$ evaluate $\iint \vec{\nabla}\times\vec{F}\cdot\hat{n}dS$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy - plane. **[1994]**

Q3. Let the region V be bounded by the smooth surface S and let n denote outward drawn unit normal vector at a point on S, if ϕ is harmonic in V, then show that $\int_S \frac{\partial\phi}{\partial n}\cdot dS$ is 0. **[1995]**

Q4. Verify Gauss's divergence theorem for $\vec{F}=xy\hat{i}+z^2\hat{j}+2yz\hat{k}$ on the tetrahedron $x=y=z=0,\ x+y+z=1$. **[1996]**

Q5. Verify Gauss's theorem for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=4,\ z=0\ \&\ z=3$. **[1997]**

Q6. Evaluate by Green's theorem: $\int_C e^{-x}\sin y\,dx+e^{-x}\cos y\,dy$, where C is rectangle whose vertices are $(0,0)$, $(\pi,0)$, $\left(\pi,\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$. **[1999]**

Q7. Evaluate $\iint_S \vec{F}\cdot\vec{n}\,ds$ where $\vec{F}=2xy\hat{i}+yz^2\hat{j}+xz\hat{k}$ and S is the surface of the parallel piped bounded by $x=0,\ y=0,\ z=0$ and $x=2,\ y=1\ \&\ z=3$. **[2000]**

Q8. Verify Gauss's divergence theorem for $\vec{A}=(4x,-2y^2,z^2)$ taken over the region bounded by $x^2+y^2=4,\ z=0\ \&\ z=3$. **[2001]**

Q9. Let D be a closed and bounded region having boundary S. Further, let f be a scalar function having second order partial derivatives defined on it. Show that

$$\iint_S (f \text{ grad } f) \cdot \hat{n}\, dS = \iiint_V \left[|\text{grad } f|^2 + f \nabla^2 f \right] dV$$

hence or otherwise evaluate: $\iint_S (f \text{ grad } f) \cdot \hat{n}\, dS$ for $f = 2x + y + 2z$ over $x^2 + y^2 + z^2 = 4$. **[2002]**

Q10. Evaluate $\iint_S \text{curl } \vec{A} \cdot d\vec{S}$, where S is the open surface $x^2 + y^2 - 4x + 4z = 0,\ z \geq 0$ and $\vec{A} = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$. **[2003]**

Q11. Derive the identity: $\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dV = \iint_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n}\, dS$, where V is the volume bounded by the closed surface S. **[2004]**

Q12. Verify Stoke's theorem for $\vec{f} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **[2004]**

Q13. Evaluate $\iint_S x^3 dydz + x^2 ydzdx + x^2 zdxdy$ by Gauss's divergence theorem, where S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by $z = 0$ & $z = b$. **[2005]**

Q14. Verify Stoke's theorem for the function: $\vec{F} = x^2\hat{i} - xy\hat{j}$, integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0,\ y = 0,\ x = a$ & $y = a,\ a > 0$. **[2006]**

Q15. Determine $\int_C ydx + zdy + xdz$ by using Stoke's theorem where C is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2,\ x + y = 2a$ that starts from the point $(2a, 0, 0)$ and goes at first below the z - plane. **[2007]**

Q16. Evaluate $\int_C \vec{A} \cdot d\vec{r}$ along the curve $x^2 + y^2 = 1,\ z = 1$ from $(0,1,1)$ to $(1,0,1)$ if $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$. **[2008]**

Q17. Evaluate $\iint_V \vec{F} \cdot \hat{n} dS$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface of the cylinder boundary by $x^2 + y^2 = 4$, $Z = 0$ and $Z = 3$. **[2008]**

Q18. Using divergence theorem, evaluate $\iint \vec{A}\cdot\hat{n}dS$ where $\vec{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$. **[2009]**

Q19. Find the value of $\iint_S \left(\vec{\nabla}\times\vec{F}\right)\cdot d\vec{S}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane $z=0$ when $\vec{F}=\left(y^2+z^2-x^2\right)\hat{i}+\left(z^2+x^2-y^2\right)\hat{j}+\left(x^2+y^2-z^2\right)\hat{k}$. **[2009]**

Q20. Use the divergence theorem to evaluate $\iint_S \vec{V}\cdot\hat{n}dA$ where $\vec{V}=x^2z\hat{i}+y\hat{j}-xz^2\hat{k}$ and S is the boundary of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4y$. **[2010]**

Q21. Verify the Green's theorem for $e^{-x}\sin y dx+e^{-x}\cos y dy$, the path of integration being the boundary of the square whose vertices are $(0,0)\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$. **[2010]**

Q22. If $\vec{u}=4y\hat{i}+x\hat{j}+2z\hat{k}$ calculate the $\iint_S \left(\vec{\nabla}\times\vec{u}\right)\cdot d\vec{S}$ over the hemisphere given by $x^2+y^2+z^2=a^2, \quad z\geq 0$. **[2011]**

Q23. Verify the Gauss's divergence theorem for the vector $\vec{u}=x^2\hat{i}+y^2\hat{j}+z^2\hat{k}$ taken over the cube $x,y,\geq 0, \qquad z^2\leq 1$. **[2011]**

Q24. Verify Green's theorem in the plane for $\oint_C \left(xy-y^2\right)dx+x^2dy$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. **[2012]**

Q25. If $\vec{F}=y\hat{i}+\left(x-2xz\right)\hat{j}-xy\hat{k}$, evaluate $\iint_S \left(\vec{\nabla}\times\vec{F}\right)\cdot\hat{n}d\vec{S}$, where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy plane. **[2012]**

Q26. By using divergence theorem of Gauss, evaluate the $\iint_S \left(a^2x^2+b^2y^2+c^2z^2\right)^{-1/2}\cdot dS$, where S is the surface of the ellipsoid $ax^2+by^2+cz^2=1$ and a,b and c being all positive constants. **[2013]**

Q27. Using Stoke's theorem to evaluate the $\int_C -y^3dx + x^2dy - z^3dz$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. **[2013, 2018]**

Q28. Evaluate by Stoke's theorem: $\int_\Gamma ydx + zdy + xdz$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the z - plane. **[2014]**

Q29. Prove that
$\oint_c d\vec{r} = \int_s \int d\vec{s} \times \nabla f$ **[2016]**

Q30. Evaluate the integral $\int_s \vec{F}.\hat{n}.\, ds$.
Where $\vec{F} = 3xy^2\hat{\imath} + (yx^2 - y^3)\hat{\jmath} + 3zx^2\hat{k}$.
and S is a surface of cylinder , $y^2 + z^2 \leq 4, -3 \leq x \leq 3$
using divergence theorem. **[2018]**

Q31. Using Green's theorem , evaluate $\int_c F(\vec{r})dr$ counter clockwise where F($\vec{r}$)= $(x^2 + y^2)\hat{\imath} + (x^2 - y^2)\hat{\jmath}$ and $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$ and the curve C is the boundary of the region R = $\{(x, y)|1 \leq y \leq 2 - x^2\}$. **[2018]**

Q32. If s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ then evaluate
$\iint_s (x + z)\, dy\, dx + (y + z)dz\, dx + (x + y)dx\, dy$ using Gauss's divergence theorem **[2018]**

Q33. Find the circulation of $\vec{F} = xy^2\hat{\imath} + (y + x)\hat{\jmath}$. Integrate $(\nabla \times \vec{F})\vec{k}$ over the region in the first quadrant bounded by the curve $y = x^2$ and $y = x$ using Green's theorem. **[2018]**

Q34. Find the circulation of $\vec{F}$ around the curve c, where $\vec{F} = (2x + y^2)\hat{\imath} + (3y - 4x)$ and c is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0). **[2019]**

Q35. (i) State Gauss divergence theorem, verify this theorem for $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **[2019]**

(ii) Evaluate by stoke's theorem $\int_c e^x dx + 2y\, dy - dz$ where c is the curve $x^2 + y^2 = 4$, z=2 **[2019]**

CIVIL SERVICES EXAMINATION (MAINS)

TUTORIAL SHEET 31: Simple Harmonic Motion (Motion in a Plane)

Q1. A particle moving with uniform acceleration describes distances S_1 and S_2 metres in successive intervals of time t_1 and t_2 seconds. Express the acceleration in terms of S_1, S_2, t_1 and t_2. **[2004]**

Q2. A particle whose mass is m, is acted upon by a force $m\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at origin in time $\frac{\pi}{4}$. **[2006, 2012]**

Q3. A particle is performing simple harmonic motion of period T about a centre O. It passes through a point $p(op = p)$ with velocity v in the direction op. Show that the time which elapses before it returns to P is $\frac{T}{\pi}\tan^{-1}\frac{VT}{2\pi p}$. **[2007]**

Q4. One end of a light elastic string of natural length l and modulus of elasticity 2 mg is attached to a fixed point O and the other end to a particle of mass m. The particle initially held at rest at O is let fall. Find the greatest extension of the string during the motion and show that the particle will reach O again after a time $\left(\pi+2-\tan^{-1}2\right)\sqrt{\frac{2l}{g}}$.

[2009]

Q5. (i) After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half of its velocity. If it now reaches the ground in 1 second, find the height of glass above the ground. **[2011]**

(ii) A particle of mass m moves on straight line under an attractive force mn^2x towards a point O on the line, where x is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find $x(t)$ for any time $t > 0$. **[2011]**

Q6. The velocity of a train increases from 0 to v at a constant acceleration f_1, then remains constant for an interval and again decreases to 0 at a constant retardation f_2. If the total distance described is x, find the total time taken. **[2011]**

Q7. A particle is performing a simple harmonic motion (SHM) of a period T about a centre O with amplitude a and it passes through a point P, where $OP = b$ in the direction of OP. Prove that the time which elapses before it returns to P is $\frac{T}{\pi}\cos^{-1}\frac{b}{a}$. **[2014]**

Q8. A particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μy^2 and when $y = a$, it is projected parallel to the axis of x with velocity $\sqrt{\frac{2m}{a}}$. Find the parametric equation of the path of the particle. Here μ is a constant. **[2014]**

Q9. A body moving under SHM has an application 'a' and time period T. If the velocity is trebled, when the distance from mean position is $\frac{2}{3}a$, the period being unaltered, Find new amplitude. **[2015]**

Q10. A particle moves in a straight line. Its acceleration is directed towards a fixed point 0 in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{\frac{1}{3}}$ when it is at a distance x from 0. If it starts from rest at a distance a from 0, then find the time, the particle will arrive at 0. **[2016]**

Q11. A particle of mass m is attached to a light wire which is stretched tightly between two fixed points with a tension T. If a,b be the distances a particle from the two ends, prove that the period of small transverse oscillation of mass m is $2\pi\sqrt{\frac{mab}{T(a+b)}}$.

Q12. A particle moving with SHM in a straight line has velocities v_1 and v_2 at distances x_1 and x_2 respectively from the centre of its path. Find the period of its motion. **[2018]**

Q13. A particle moving along the y-axis has an acceleration Fy towards the origin where F is a positive and even function of y. The periodic time, when the particle vibrate between $y = -a$ and $y = a$, is T, show that $\frac{2\pi}{\sqrt{F_1}} < T < \frac{2\pi}{\sqrt{F_2}}$ where F_1 and F_2 are the greatest and least values of F within the range $[-a, a]$. Further show that were a simple pendulum of length & oscillates through 30° on either side of vertical line, T lies between $2\pi\sqrt{\frac{l}{g}}$ and $2\pi\sqrt{\frac{l}{g}}\sqrt{\frac{\pi}{3}}$ **[2019]**

CIVIL SERVICES EXAMINATION (MAINS)

TUTORIAL SHEET 32: Projectile Motion

Q1. Prove that the velocity required to project a particle from a height h to fall at a horizontal distance a from a point of projection is at least equal to $\sqrt{g\sqrt{a^2+h^2}-h}$. **[2004]**

Q2. If V_1, V_2, V_3 are the velocities at three points A, B, C of the path of a projectile, where the inclinations to the horizon are $\alpha, \alpha-\beta, \alpha-2\beta$ and if t_1, t_2 are the times of describing the arcs AB, BC respectively, prove that $V_3t_1 = V_1t_2$ and $\frac{1}{V_1}+\frac{1}{V_3}=\frac{2\cos\beta}{V_2}$. **[2010]**

Q3. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a meter short of it when the angle of projection is α and goes y meter beyond when the angle of projection is β. If the velocity of projection is assumed same in all cases, find the correct angle of projection. **[2011]**

Q4. A particle is free to move on a smooth vertical circular wire of radius *a*. At time *t*= 0, it is projected along the circle from the lowest point A with velocity just sufficient to carry it to the highest point B. Find the time *T* at which the reaction between the particle and wire is zero. **[2017]**

Q5. A particle projected from a given point on the ground just clears a wall of height *h* at a distance *d* from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R. Find the elevation of the projection. **[2018]**

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CIVIL SERVICES EXAMINATION (MAINS)

TUTORIAL SHEET 33: Constrained Motion

Q1. If a particle slides down a smooth cycloid, starting from a point whose actual distance from the vertex is b, prove that its speed at any time t is $\frac{2xb}{T}\sin\left(\frac{2xT}{T}\right)$, where T is the time of complete oscillation of the particle. **[2003]**

Q2. A particle is projected along the inner side of a smooth vertical circle of radius a so that velocity at the lowest point is u. Show that $2ag < u^2 < 5ag$. The particle will the highest point and will describe a parabola whose latus rectum is $\frac{2\left(u^2-2ag\right)^3}{27a^2g^3}$.

[2005]

Q3. Two particles connected by a fine string are constrained to move in a fine cylindrical tube in a vertical plane. The axis of the cycloid is vertical with vertex upwards. Prove that the tension in the string is constant throughout motion. **[2005]**

Q4. A particle is free to move on a smooth vertical circular wire of radius a. It is projected horizontally from the lowest point with velocity $2\sqrt{ag}$. Show that the reaction between the particle and the wire is zero after a time $\sqrt{\frac{a}{g}}\log\left(\sqrt{5}+\sqrt{6}\right)$. **[2006]**

Q5. A particle is projected with velocity v from the cusp of a smooth inverted cycloid down the arc. Show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\cot^{-1}\frac{v}{2\sqrt{ag}}$. **[2009]**

Q6. A particle is free to move on a smooth vertical circular wire of radius a. At time t=0 , it is projected along the circle from its lowest point A with velocity just sufficient to carry it to the height point B . Find the time T at which the reaction between the particles and the wire is zero. **[2017]**

TUTORIAL SHEET 34: Central Orbits

Q1. A particle of mass m moves under a force $m\mu\left\{3au^4 - 2\left(a^2 - b^2\right)u^5\right\}$, $u = \frac{1}{r}$, $a > b$ and $\mu > 0$ being given constants. It is projected from as apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is given by $r = a + b\cos\theta$, where $\left(r, \theta\right)$ are the plane polar coordinates of a point. **[2008]**

Q2. A body is describing an ellipse of eccentricity e under the action of a central force directed towards a focus and when at the nearer apse, the centre of force is transferred to other focus. Find the eccentricity of the new orbit in terms of the original orbit. **[2009]**

Q3. A particle moves with a central acceleration $\mu\left(r^5 - 9r\right)$ being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{2u}$. Show that its path is $x^4 + y^4 = 9$. **[2010]**

Q4. A particle moves in a plane under a force, towards a fixed centre, proportional to the distance. If the path of the particle has 2 apsidal distance a, b (a>b), then find the equation of the path. **[2015]**

Q5. A man starts from rest at a distance a from the centre of force which attracts inversely as the distance. Find the time of arriving at the centre. **[2015]**

Q6. A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is projected from and apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, then find the equation to the path. **[2016]**

Q7. Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to $\frac{\mu}{(distance)^2}$, is a conic section. Find the condition under which the path becomes (i) ellipse (ii) parabola (iii) hyperbola. **[2019]**

TUTORIAL SHEET 35: Work, Energy and Impulse

Q1. A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $=\sqrt{\dfrac{2mE}{M(M+m)}}$ and the initial velocity of the shot is $\sqrt{\dfrac{2ME}{M(M+m)}}$.

Q2. A shell of mass M is moving with velocity v. An internal explosion generates an amount of energy E and breaks the shell into two portions whose masses are in the ratio $m_1 : m_2$. The fragments continue to move in the original line of motion of the shell. Show that their velocities are $v+\sqrt{\dfrac{2m_2E}{m_1M}}$ and $v-\sqrt{\dfrac{2m_1E}{m_2M}}$.

Q3. A bullet of mass m moving with velocity v, strikes a block of mass M, which is free to move in the direction of the motion of the bullet and is embedded in it. Show that a portion $\dfrac{M}{M+m}$ of the K.E. is lost. If the block is afterwards struck by an equal bullet moving in the same direction with the same velocity. Show that there is a further loss of K.E. equal to $\dfrac{mM^2v^2}{2(m+M)(M+2m)}$.

Q4. A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Prove that the velocity of the recoil is $\left[\dfrac{2m^2gh}{M(M+m)}\right]^{1/2}$.

Q5. A train of mass M lb is ascending a smooth incline of 1 in n and when the velocity of the train is v ft/sec, its acceleration is f ft/sec^2. Prove that the effective HP of the engine is $\dfrac{Mv(nf+g)}{550\,ng}$.

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Q6. The force of attraction of particle by the earth is inversely proportional to the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $3h$ above it. Show that the magnitude of work done by the earth's attraction force is $\frac{3}{4}hW$, where h is radius of the earth. **[2019]**

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TUTORIAL SHEET 36.Equilibrium of system of particle, Principle of virtual work

Q1. If a number of concurrent force be represented in magnitude and direction by the side of a closed polygon, taken in orders, then show there forces are in equilibrium.
[2005]

Q2. The middle point of opposite sides of joined quadrilateral are connected by light rods of length l, l'. If T, T' be the tension in these rods, prove that $\frac{T}{l} + \frac{T'}{l'} = 0$ **[2006]**

Q3. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with the curved surface of the hemisphere is in contact. If θ and $\emptyset$ are the inclination of the string and plane base of the hemisphere to the vertical, prove by using the principle of virtual work tan $\emptyset = \frac{3}{8} + tan\theta$. **[2010]**

Q4. Six equal rod AB, BC, CD, DE, EF & FA are each of weight w and are freely joined at their extremities so as to form a hexagon, rod AB is fixed on a horizontal position and middle point of AB & DE are joined by a string find the tension in string.

Q5. Two equal uniform rods AB & AC each of length l, are freely joined at A and rest on a smooth fixed vertical circle of radius r. if 2θ is the angle between the rods, then find relations between l, r, θ and using principle of virtual work. **[2014]**

Q6. A regular pentagon ABCDE formed of equal heavy uniform bars joined together is suspended from joint A and is maintained in form by a light rod joining the middle point of BC & DE. Find stress in this rod. **[2014]**

Q7. A rod of length 8 kg is movable in vertical plane about a hinge at one end, another end is fastened a weight equal to half of rod, this end is fastened by a string of length l to appoint at a weight b above the hinge vertically obtain tension in the string. **[2015]**

Q8. A uniform rod AB of length 2a movable about a hinge at A rests with other end against a smooth vertical wall. If α is inclination of rod to vertical prove that magnitude of reaction of hinge is $\frac{1}{2}w\sqrt{4 + tan^2\alpha}$ where w is the weight of the rod. **[2016]**

Q9. Two weight P & Q are suspended from a fixed point O by strings OA, OB and are kept apart by a light rod AB. If the string OA & OB makes an angle α and β with the rods AB, show that the angle θ which the rod makes with the vertical is given by

$tan\theta = \frac{P+Q}{P\cot\alpha - Q\, cot\,\beta}$ **[2016]**

Q10. A square ABCD, the length of whose side is a, is fixed in vertical plane with two of its sides horizontal. An endless string of length $l\ (> 4a)$ passes over four pegs at the angle of the board and through a ring of weight w which is hanging vertically. Show tension in a string is $\frac{w(l-3a)}{2\sqrt{l^2-6a=8a^2}}$ **[2016]**

TUTORIAL SHEET 37. Work & Potential energy & fiction

Q1. A straight uniform bean of length '2h' rests in limiting equilibrium, in contact with a rough vertical wall of height 'h' with me end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall and the plane be equally rough, prove that $'\lambda'$ the angle of friction is fiven by $2\lambda = \sin\alpha \sin 2\alpha$, α being the inclination of beam to the horizon. **[2008]**

Q2 A heavy ring of mass m slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distant a from the rod and has a mass $M(> m)$ fastened to other end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley it will descend a distance $\frac{2Mma}{M^2-m^2}$, before coming to rest.

[2012]

Q3. The base of an inclined plane is 4m in length and height 3 metres. A force of 8 kg acting parallel to plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of fiction between the plane and weight. **[2013]**

Q4. A uniform ladder rests at an angle of 45° with horizontal with its upper extremity against a rough vertical wall and its lower extremity on ground. If u and u' the coefficient of limiting fiction between the ladder and ground & wall respectively then find minimum horizontal force required to move the lower end of ladder towards the wall. **[2013]**

Q5. Two equal ladders of weight 4 kg each are placed so as to lean at A against each other with their ends resting on a rough floor, given the coefficient of friction is u. The ladders at A make an angle 60° with each other. Find what weight on top would cause them to slip. **[2015]**

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Q6. One end of heavy uniform rod AB can slide along a rough horizontal rod AC to which it is attached by a ring B and C are joined by a string. When the rod is on the point of sliding them then $AC^2 - AB^2 = BC^2$. If θ be angle between AB and the horizontal line then prove that coefficient of friction is $\frac{\cot\theta}{2+cot^2\theta}$ **[2019]**

TUTORIAL SHEET 38 Common Catenary

Q1. Show that the length of an endless chain which will hang over a circular pulley of radius c so as to be in contact with $^2/_3$ of the circumference of the pulley is

$c\left\{\frac{3}{\log(2+v3)}+\frac{4\pi}{\sqrt{3}}\right\}$ **[2006]**

Q2. A uniform string of length 1m hangs over two smooth pegs P and Q at different heights. The parts which hang vertically are of length 34 cm and 26 cm. Find the ratio in which the vertex of the catenary divides the whole string. **[2007]**

Q3. Find the length of an endless chain which will have over a circular pulley of radius a so as to be in contact with $\frac{3}{4}th$ of the circumference of the pulley. **[2009]**

Q4. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$u\log\left[\frac{1+\sqrt{1+u^2}}{u}\right]$ **[2012]**

TUTORIAL SHEET 39 Stability of equilibrium & Equilibrium of forces in 3 dimensions

Q1. A uniform beam of length l rests with its ends or two smooth planes which intersect in a horizontal line. If the inclinations of the planes to the horizontal are α and β $(\beta > \alpha)$. Show that the inclination θ
Of the beam to the horizontal in one of the equilibrium positions is given by
$tan\theta = \frac{1}{2}(cot\alpha - cot\beta)$ & show that the beam is unstable in this position.

[2007]

Q2. A heavy hemispherical shell of radius a has a particle attached to a point on the rim with a rough sphere of radius b at the highest point. Prove that if $\frac{b}{a} > \sqrt{5} - 1$ the equilibrium is stable whatever be the weight of particle. **[2012]**

Q3 A uniform solid hemisphere rests on a rough plane inclined l the horizon at an angle $\emptyset$ with its curved surface touching the plane. Find the greatest admissible value of the inclination $\emptyset$ for equilibrium. If $\emptyset$ be less than this value, is the equilibrium stable?

[2017]

Q4 A body consists of a cone and underlying hemisphere. The base of the cone and the top of hemisphere have same radius a. The whole body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}a$. **[2019]**