CIVIL SERVICES EXAMINATION (MAINS)

Mathematics Paper – II: Section B

A: Partial Differential Equations **(10 Classes)**

Unit	Content	Lectures
Unit 18:	Linear Partial Differential Equations of Order 1	**(2 Classes)**
Unit 19:	Partial Differential Equations of Degree more than 1, Charpit's Method	**(2 Classes)**
Unit 20:	Partial Differential Equations of 2nd Order with constant coefficients	**(2 Classes)**
Unit 21:	Applications of Partial Differential Equations to vibrating strings, Heat and Laplace Equations.	**(3 Classes)**
Unit 22:	Canonical form and Miscellaneous Problems	**(1 Class)**

B: Numerical Analysis & Computer Programming **(10 Classes)**

Contents **Lectures**

Unit 23: Solution of Algebraic and Transcendental Equations of One variable **(1 class)**

- Bi-Section Method
- Regula-Falsi Method
- Newton-Raphson Method

Unit 24: Solution of System of Linear Equations **(1 class)**

- Gaussian Elimination Method
- Gauss-Jordan Method (Direct)
- Gauss-Seidel (Iterative) Method

Unit 25: Interpolation **(2 classes)**

- Newton's (forward & backward) formulas
- Lagrange's Method

Unit 26: Numerical Integration **(2 classes)**

- Trapezoidal Rule
- Simpson's Rule
- Gaussian Quadrature Formula

Unit 27: Numerical Solutions of Differential Equations **(2 classes)**

- Euler's Method
- Runge-Kutta Methods

Unit 28: Number Systems, Boolean Algebra and Flow Charts **(2 classes)**

- Different Number Systems and Inter-conversions
- Boolean Identities
- Logic Gates
- Algorithms and Flow Charts for solving problems in Numerical Analysis

C: Mechanics and Fluid Dynamics **(20 Classes)**

Unit	**Contents**	**Lectures**
Unit 29:	Kinematics	**(2 Classes)**
Unit 30:	Euler's Equation of Motion	**(2 Classes)**
Unit 31:	Motion in 2 Dimensions Potential Flow, Sources and Sinks	**(2Classes)**
Unit 32:	Axisymmetric Motion(Motion of Cylinders and Spheres)	**(2Classes)**
Unit 33:	Vortex Motion	**(2 Classes)**
Unit 34:	Navier Stokes Equations for a viscous fluid	**(2 Classes)**
Unit 35:	Moment of Inertia	**(2 Classes)**
Unit 36:	Lagrange's Equation of Motion	**(2 Classes)**
Unit 37:	Hamiltonian Equation of Motion	**(2 Classes)**
Unit 38:	De Alembert's Principle / Equations of Motion in 2 Dimensions	**(2 Classes)**

CIVIL SERVICES EXAMINATION (MAINS)

MATHEMATICS PAPER II: Partial Differential Equations

Unit 18: Framing of P.D.E. and Linear P.D.E. of Order One

Q1. Find the integral surface of the following P.D.E. $x\left(y^2+z\right)p-y\left(x^2+z\right)q=\left(x^2-y^2\right)z$.

(2004)

Q2. Formulate P.D.E. for surface whose tangent planes form a tetrahedron of constant volume with coordinate planes. **(2005)**

Q3. Find the particular integral of $x(y-z)p+y(z-x)q=z(x-y)$. **(2005)**

Q4. Solve: $px\left(z-2y^2\right)=(z-qy)\left(z-y^2-2x^3\right)$ **(2006)**

Q5. Form a P.D.E. by eliminating the function f from $z=y^2+2f\left(\frac{1}{x}+\log y\right)$. **(2007)**

Q6. Solve $2zx-px^2-2qxy+pq=0$. **(2007)**

Q7. Find the general solution of P.D.E. $(2xy-1)p+\left(z-2x^2\right)q=2(x-yz)$ and also find the particular solution which passes through the lines $x=1,\ y=0$. **(2008)**

Q8. Form the P.D.E. by eliminating the arbitrary function f given by $f\left(x^2+y^2,\ z-xy\right)=0$.

(2009)

Q9. Show that D.E. of all cones which have their vertex at the origin is $px+qy=z$. Verify that this equation is satisfied by the surface $yz+zx+qy=0$. **(2009)**

Q10. Solve the P.D.E. $(x+2z)p+(4xz-y)q=2x^2+y$. **(2011)**

Q11. Solve the P.D.E. $px+qy=3z$. **(2012)**

Q12. Form a P.D.E. by eliminating the arbitrary functions f and g from $z=y\,f(x)+x\,g(y)$.

(2013)

Unit 19: P.D.E. of degree more than one (Charpit's Method)

Q1. Using Charpit method, find the complete internal of $2xz - px^2 - 2qxy + pq = 0$. **(1993)**

Q2. Use Charpit's method to solve $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$. Interpret geometrically the complete solution and mention the singular solution. **(1994)**

Q3. Explain in detail the Charpit's method of solving the nonlinear P.D.E. **(1995)**

Q4. Solve $z^2\left(p^2 + q^2 + 1\right) = c^2$. **(1996)**

Q5. Solve by Charpit's method: $z = px + qy + p^2 + q^2$. **(1996)**

Q6. Solve by Charpit's method: $z^2\left(p^2z^2 + q^2\right) = 1$. **(1997)**

Q7. Use Charpit's method to find complete integral of $\left[2x\left(z\frac{\partial z}{\partial y}\right)^2 + 1\right] = z\frac{\partial z}{\partial x}$. **(1998)**

Q8. Use Charpit's method to find complete integral of $p^2 + q^2 - 2px - 2qy + 1 = 9$. **(1999)**

Q9. Solve by Charpit's method: $p^2x(x-1) + 2pq\,xy + q^2y(y-1) - 2pxz - 2qyz + z^2 = 0$. **(2000)**

Q10. Solve $2p^2q^2 + 3x^2y^2 = 8x^2q^2\left(x^2 + y^2\right)$. **(2001)**

Q11. Solve the equation $p^2 - q^2 - 2px - 2qy + 2xy = 0$ using Charpit's method also find the singular solution of the equation if it exists. **(2003)**

Q12. Using Charpit's method, find the complete solution of the P.D.E. $p^2x + q^2y = z$. **(2004)**

Q13. Solve by Charpit's method $p^2x + q^2y = z$. **(2006)**

Q14. Find the complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method. **(2008)**

Unit 20: Linear partial Differential Equations of 2nd Order with Constant Coefficients

Q1. Find the general solution of

$$\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y+\cos(2x+3y).$$ **(2003)**

Q2. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}-3\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=xy+e^x+2y$. **(2003)**

Q3. Solve the P.D.E. $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=(y-1)e^x$. **(2004)**

Q4. Obtain the general solution of $(D-3D-2)^2 z=2e^{2x}\sin(y+3x)$. **(2005)**

Q5. Solve: $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial x\partial y^2}=2\sin(3x+2y)$. **(2006)**

Q6. Find the general solution of $\left(D^2+DD'-6D'^2\right)z=y\cos x$. **(2008)**

Q7. Solve: $\left(D^2-DD'-2D'^2\right)z=\left(2x^2+xy-y^2\right)\sin xy-\cos xy$. **(2009)**

Q8. Solve the P.D.E. $\left(D^2-D'\right)\left(D-2D'\right)z=e^{2x+y}+xy$. **(2010)**

Q9. Find the surface satisfying the P.D.E. $\left(D^2-2DD'+D'^2\right)z=0$ and the conditions that $bz=y^2$ when $x=0$ and $az=x^2$ when $y=0$. **(2010)**

Q10. Solve the P.D.E. $\left(D^2-D'^2+D+3D'-2\right)z=e^{x-y}-x^2y$. **(2011)**

Q11. Solve the P.D.E. $\left(D-2D'\right)\left(D-D'\right)^2 z=e^{x+y}$. **(2012)**

Q12. Solve: $\left(D^2+DD'-6D'^2\right)z=x^2\sin(x+y)$. **(2013)**

Q13. Solve the Partial Differential Equation $\left[2D^2-5DD'+2D'^2\right]z=24(y-x)$. **(2014)**

Unit 21: Application of P.D.E. (Vibrating String, Heat Equation & Laplace Equation)

Q1. Find the deflection $u(x,t)$ of a vibrating string, stretches between fixed points $(0,0)$ and $(3l,0)$, corresponding to zero initial velocity and following initial deflection

$$f(x)=\frac{hx}{l} \quad \text{where } 0\le x\le l$$

$$=\frac{h(3l-2x)}{l} \quad \text{where } l\le x\le 2l$$

$$=\frac{h(x-3l)}{l} \quad \text{where } 2l\le x\le 3l$$

(2003)

Q2. A uniform string of length l held tightly between $x=0$ and $x=l$ with no initial displacement, is struck at $x=a$, $0<a<l$ with velocity v_0. Find the displacement of the string at any time $t>0$. **(2004)**

Q3. The ends A and B of a rod 20 cm long have the temperature at $30^0 C$ and at $80^0 C$ until steady state prevails. The temperature of the ends are changed to $40^0 C$ and $60^0 C$ respectively. Find the temperature distribute in the rod at time t. **(2005)**

Q4. The deflection of a vibrating string of length l is governed by P.D.E. $u_{tt}=c^2u_{xx}$. The ends of the string are fixed at $x=0$ and l. The initial velocity is zero. The initial displacement is given by

$$u(x,0)=\frac{x}{l} \qquad 0<x<\frac{l}{2}=\frac{l-x}{l}\ \frac{l}{2}<x<l$$

Find the deflection of the string at any instant of time. **(2006)**

Q5. Solve $u_{xx}+u_{yy}=0$ in D, where $D:\{(x,y):0<x<a,\ 0<y<b\}$ is a rectangle in a plane with the boundary conditions

$$u(x,0)=0 \qquad u(x,b)=0 \qquad 0\le x\le a$$

$$u(0,y)=g(y)ux(a,y)=h(y) \qquad 0\le y\le b.$$

(2007)

Q6. Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions $u(0,t) = 0 = u(l,t)$ for all t and $u(x,0) = f(x)$ for all x in $[0,l]$. **(2007)**

Q7. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0,\ x = a,\ y = 0,\ y = b$. The edges $x = 0,\ x = a$ and $y = 0$ are kept at temperature zero while the edge $y = b$ is kept at $100^0 C$. **(2008)**

Q8. A tightly stretched string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant and then released. Find the displacement at any point x of the string at time $t > 0$. **(2009)**

Q9. Solve the following heat equation:

$$u_t - u_{xx} = 0 \qquad 0 < x < 2; \qquad t > 0$$

$$u(0,t) = u(2,t) = 0; \qquad t > 0$$

$$u(x,0) = x(2 - x) \qquad 0 \le x \le 2$$

(2010)

Q10. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $\qquad 0 < x \le a$, $0 < y \le b$

Satisfying the boundary conditions

$$u(0,y) = 0, \quad u(x,0) = 0, \quad u(x,b) = 0$$

$$\frac{\partial u}{\partial x}(a,y) = T \sin^3 \frac{\pi y}{a}$$

(2011)

Q11. Obtain temperature distribution $y(x,t)$ in a uniform bar of unit length whose one end is kept at $10^0 C$ and the other end is insulated. Also it is given that $y(x,0) = 1 - x \qquad 0 < x < 1$ **(2011)**

Q12. A string of length l is fixed at its ends. The string from the mid point is pulled up to a height K and then released from rest. Find the deflection $y(x,t)$ of vibrating string.

(2012)

Q13. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. **(2012)**

Q14. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving ends point a velocity $\lambda(x)(l - x)$. Find the displacement of the string at any distance x from one end at any time t. **(2013)**

Q15. Find the deflection of a vibrating string (length $= \pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$). Corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$.

(2014)

Q16. Solve: $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $\quad 0 < x < 1$,

Given that

(i) $u(x,0) = 0 \qquad 0 \le x \le 1$

(ii) $\frac{\partial u}{\partial t}(x,0) = x^2; \qquad 0 \le x \le 1$

(iii) $u(0,t) = u(1,t)$ for all t. **(2014)**

Unit 22: Canonical forms/Cauchy's P.D.E. and Miscellaneous Problems

Q1. Show that the differential equation of all cones which have their vertex at the origin are $px+qy=z$. Verify that $yz+zx+xy=0$ is a surface satisfying the above equation. **(2003)**

Q2. Find the integral surface of the following P.D.E. $x\left(y^2+z\right)p-y\left(x^2+z\right)q=\left(x^2-y^2\right)z$. **(2004)**

Q3. Find the complete integral of the P.D.E. $\left(p^2+q^2\right)x=pz$ and deduce the solution which passes through the curve $x=0, z^2=4y$. **(2004)**

Q4. Find the surface passing through the parabolas $z=0, y^2=4ax$ and $z=1, y^2=-4ax$ and satisfying the equation $x\dfrac{\partial^2 z}{\partial x^2}+2\dfrac{\partial z}{\partial x}=0$. **(2006)**

Q5. Transform the equation $yz_x-xz_y=0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of resolution. **(2007)**

Q6. Reduce $\dfrac{\partial^2 z}{\partial x^2}=x^2\dfrac{\partial^2 z}{\partial y^2}$ to canonical form. **(2008, 2014)**

Q7. Find the characteristics of $y^2r-x^2t=0$, where r and t have their usual meanings. **(2009)**

Q8. Find the integral surface of $x^2p+y^2q+z^2=0$ which passes through the $xy=x+y,\ z=1$. **(2009)**

Q9. Reduce the following equations to canonical form and fixed its general solution $xu_{xx}+2xu_{xy}-u_x=0$. **(2010)**

Q10. Solve the following P.D.E. by the method of characteristics:

$$zp+yq=x$$

$$x(\dot{s})=s, \qquad y_0(s)=1, \qquad z_0(s)=2s.$$ **(2010)**

Q11. Find the surface satisfying $\dfrac{\partial^2 z}{\partial x^2}=6x+2$ and trouching $z=x^3+y^3$ along its section by the plane $x+y+1=0$. **(2011)**

Q12. Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1), C$ is constant, orthogonally and which passes through the circle $x^2+y^2=1,\ z=1$. **(2013)**

Q13. Reduce the equation to canonical form when $x \neq y$

$$y\frac{\partial^2 z}{\partial x^2}+(x+y)\frac{\partial^2 z}{\partial x \partial y}+x\frac{\partial^2 z}{\partial y^2}.$$ **(2014)**

Mathematics Paper II: Section B

Numerical Analysis & Computer Programming

Unit 23: Solution of Algebraic and Transcendental equations.

Q1. Find a root of the equation $x\sin x+\cos x=0$ using Newton-Rapson Method. **(1988)**

Q2. The polynomial x^3-x-1 has a root between 1 and 2. Using the Secant method, find this root correct to 3 significant figures. **(1989)**

Q3. Solve $x^2-5x+3=0$ in the interval $[1,2]$ by Secant Method. **(1990)**

Q4. Using Regula-Falsi method, find the real root of the equation $x\log_{10}x-1.2=0$, correct to 5 decimal places. (Ans. 2.74064844) **(1991)**

Q5. Compute to 4 decimal places by using Newton-Raphson method, the real root of

$$x^2+4\sin x=0$$ **(1992)**

Q6. Find correct to 3 decimal places the real root of $2e^x-3x^2=2.5644$. **(1993)**

Q7. Find the positive root of $\log_e x=\cos x$, nearest to 5 decimal places by Newto-Raphson method. **(1995)**

Q8. Describe Newton-Raphson method for finding the solutions of the equation $f(x)=0$ and show that method has a quadratic convergence. **(1996)**

Q9. Show that the iteration formula for finding the reciprocal of N is $x_{n+1}=x_n(2-N_{xn}),\ \ n=0,1,....$ **(1997)**

Q10. Use Regula-Falsi method to show that the real root of $x\log_{10}x-1.2=0$ lies between 3 and 2.740646. **(1998)**

Q11. Find the positive root of the equation $2e^{-x}=\frac{1}{x+2}+\frac{1}{x+1}$. Using Newton-Raphson method correct to 4 decimal places. Also show that the following scheme has error of second order

$$x_{n+1}=\frac{1}{2}x_n\left(1+\frac{a}{x_n^2}\right).$$ **(2003)**

Q12. How many positive and negative roots of the equation $e^x-5\sin x=0$ exist? Find the smallest positive root correct to 3 decimals, using Newton-Raphson method. **(2004)**

Q13. Use the method of false position to find a real root of $x^3 - 5x - 7 = 0$ lying between 2 and 3 and correct to 3 places of decimals. **(2007)**

Q14. Develop at algorithm for Regula-Falsi method to find a root of $f(x) = 0$ starting with two initial iterates x_0 and x_1 to the rot such that $f(x_0) \neq f(x_1)$. Take n as the maximum number of iteration allowed and eps be the prescribed error. **(2010)**

Q15. Find the positive root of the equation $10xe^{-x^2} - 1 = 0$ correct upto 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **(2010)**

Q16. Use Newton-Raphson method to find the real root of $3x = \cos x + 1$ correct to 4 decimal places. **(2012)**

Q17. Develop an algorithm for Newton-Raphson method to solve $f(x) = 0$. **(2013)**

Q18. Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct upto 4 decimal places. **(2014)**

Unit 24: INTERPOLATION

Q1. Certain corresponding values of x and $\log x$ are given

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4847

Find $\log_{10} 301$, using Lagrange's interpolation formula. **(1988)**

Q2. Apply Lagrange's formula to find a root of the equation $f(x)=0$ given that

$f(30)=-30,\quad f(34)=-13,\quad f(38)=3,\quad f(42)=18$. **(1991)**

Q3. The following are the measurements t made on a curve recorded by the oscillograph representing a change of current i due to a change in the condition of an electric current.

t	1.2	2.0	2.5	3.0
i	1.36	0.58	0.34	0.20

Applying an appropriate formula interpolate for the value of i when $t=1.6$. **(1996)**

Q4. Find the cubic polynomial which takes following values:

$y(0)=1,\quad y(1)=0,\quad y(2)=1$ & $y(3)=10$

Hence, otherwise $y(4)$. **(2002)**

Q5. Find the unique polynomial $P(x)$ of degree 2 or less such that $P(1)=1$, $P(3)=27$, $P(4)=64$. Using the Lagrange's interpolation formula and the Newton's divided difference formula evaluate $P(15)$. **(2005)**

Q6. The following values of the function $f(x)=\sin x+\cos x$ are given

x:	10	20	30
$f(x)$:	1.1585	1.2817	1.3360

Construct the quadratic interpolating polynomial that fits the data. Hence calculate $f\left(\frac{\pi}{12}\right)$. Compare with exact value.

Q7. Value of $f(3)$ from the following table:

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

(2009)

Q8. Draw a flow chart for Lagrange's interpolation.

Q9. In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. **(2013)**

Unit 25: Gauss Elimination / Gauss-Seidel Method

Q1. Solve the following system of linear equations using Gauss elimination method:

$$x_1 + 6x_2 + 3x_3 = 6$$

$$2x_1 + 3x_2 + 3x_3 = 117$$

$$4x_1 + x_2 + 2x_3 = 283.$$ **(2000)**

Q2. Using Gauss-Seidel iterative method and the starting solution as $x_1 - x_2 - x_3 = 0$, determine the solution of the following system of equations in two iterations:

$$10x_1 - x_2 - x_3 = 8$$

$$x_1 + 10x_2 + x_3 = 12$$

$$x_1 - x_2 + 10x_3 = 10$$

Q3. Using Gauss-Seidel iteration method, find the solution of the following system upto three iteration:

$$4x - y + 8z = 26$$

$$5x + 2y - z = 6$$

$$x - 10y + 2z = -13.$$ **(2004)**

Q4. Apply Gauss-Seidel method to calculate x, y, z from the system

$$-x - y + 6z = 42$$

$$6x - y - z = 11.33$$

$$-x + 6y - z = 32$$

with initial values (4.67, 7.62, 9.05). Carry out computations for two iterations. **(2008)**

Q5. Solve using Gauss-Seidel method:

$$3x + 20y - z = -18$$

$$20x + y - 2z = 17$$

$$2x - 3y + 20z = 25.$$ **(2012)**

Q6. Solve the system of equations

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1$$

Using Gauss-Seidel iteration method (perform three iteration). **(2014)**

Unit 26: Numerical Integration

Q1. Use the method of Gauss to verify the following computation

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1+\sin^2\theta}} = 1.311028. \qquad \textbf{(1990)}$$

Q2. Evaluate approximately $\int_{-3}^{3} x^4 dx$ by Simpson's rule by taking seven equidistant ordinates. Compare it with the value obtained by trapezoidal rule and with exact value. **(1993)**

Q3. Find the value of $\int_{1.6}^{3.4} f(x)dx$ from the following data using Simpson's 3/8th rule for the interval (1.6, 2.2) and 1/3rd rule for (2.2, 3.4)

x	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4
$f(x)$	4.953	6.050	7.389	9.025	11.023	13.464	16.445	20.086	24.533	29.964

Q4. Evaluate $\int_1^3 \frac{dx}{x}$ by Simpson's rule with 4 strips. Determine the error by direct integration.

Q5. Obtain Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \le 3$. Show that the error of approximation for Simpson's rule is given by

$$R = -\frac{(b-a)}{2880} f^{iv}(\eta) \quad \eta \in (0,2)$$

Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \le 0.008333$. **(1999)**

Q6. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by subdividing the interval $(0,1)$ into 6 equal parts and using Simpson's 1/3rd rule. Hence find the value of π and actual error, correct to five places of decimal. **(2000)**

Q7. Evaluate $\int_0^1 e^{-x^2} dx$ employing three point Gaussian quadrature formula, finding the required weights and residues. Use five decimals for computation. **(2003)**

Q8. Draw a flow chart and write a program in basic for Simpson's 1/3rd rule for integration $\int_a^b \frac{dx}{1+x^2}$ correct upto 10^{-6}. **(2003)**

Q9. The velocity of a particle at distances from a point on its path is given by the following table:

S meter	0	10	20	30	40	50	60
V m/sec	47	58	64	65	61	52	38

Estimate the time taken to travel the first 60 meters using Simpson's 1/3rd rule. Compare the result with Simpson's 3/8th rule. **(2004)**

Q10. Use appropriate quadrature formula out of Trapezoidal and Simpson's rules to numerically integrate $\int_0^1 \frac{dx}{1+x^2}$ with $h = 0.2$. Hence obtain an approximate value of π. Justify the use of particular quadrature formula. **(2005)**

Q11. Evaluate $\int_0^1 e^{-x^2} dx$ by Simpson's 1/3rd rule with $2\pi = 10,\ x_0 = 0,\ h = 0.1,\ x_{10} = 1.0$. **(2006)**

Q12. Find from the following table, the area bounded by the x- axis and the curve $y = f(x)$ between $x = 5.34$ and $x = 5.4$ using Trapezoidal rule.

x	5.34	5.35	5.36	5.37	5.38	5.39	5.40
$f(x)$	1.82	1.85	1.86	1.90	1.95	1.97	2.00

Q13. Evaluate $\int_1^5 \log_{10} x dx$ by Simpson's 1/3rd rule correct upto 4 decimal places. Take 8 sub intervals in your computation. **(2010)**

Q14. A solid of revolution is formed by rotating about the x- axis, the area between the x- axis, the line $x = 0$ and $x = 1$ and a curve through the points with the following coordinates:

x	0.0	0.25	0.50	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Find value of the solid. **(2011)**

Q15. The velocity of a train which starts from rest given in the following table

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using Simpson's 1/3rd formula. **(2013)**

Q16. Use five subintervals to integrate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule. **(2014)**

Unit 27: Numerical Solution of Ordinary Differential Equations

Q1. Use Runge-Kutta method to solve

$$10\frac{dy}{dx} = x^2 + y^2, \qquad y(0) = 1$$

for the interval $0 < x \le 0.4$ with $h = 0.1$. **(1989)**

Q2. Using Runge-Kutta method with 3rd order accuracy solve $\frac{dy}{dx} = y - x$ with initial condition $y = 2,\ x = 0$. **(1990)**

Q3. Solve by Runge-Kutta method $\frac{dy}{dx} = x + y$ with the initial conditions $x_0 = 0$, $y_0 = 1$ correct upto 4 decimal places by evaluating upto second increment of y. (Take $h = 0.1$). **(1992)**

Q4. Solve $\frac{dy}{dx} = xy$ for $x = 1.4$ by Runge-Kutta method, initially $x = 1,\ y = 2$. (Take $h = 0.2$). **(1993)**

Q5. Apply that 4th order Runge-Kutta method to find value of y correct to 4 places of decimals at $x = 0.2$, when $y' = \frac{dy}{dx} = x + y,\ y(0) = 1$. **(1997)**

Q6. By the 4th order Runge-Kutta method, tabulate the solution of Differential Equation

$$\frac{dy}{dx} = \frac{xy + 1}{10y^2 + 4}, \qquad y(0) = 0$$

in [0, 0.4] with step length of 0.1 correct to 5 places of decimal. **(1998)**

Q7. Given $\frac{dy}{dx} = 1 + y^2$, where $y = 0$ and $x = 0$, find $y(0.2)$, $y(0.4)$ and $y(0.6)$.

Q8. Using 4th order classical Runge-Kutta method for the initial value problem

$$\frac{du}{dt} = -2tu^2 \qquad u(0) = 1$$

with $h = 0.02$ on the interval [0, 1]. Calculate $u(0.4)$ correct to six places of decimal. **(1999)**

Q9. Runge-Kutta 4^{th} order method, find $y(0.1)$ and $y(0.2)$. Compare the approximate solution with its exact solution. $e^{0.1} = 1.10517,\ e^{0.2} = 1.2214$. **(2002)**

Q10. Apply the 2^{nd} order Runge-Kutta method to find an approximate value of y at $x = 0.2$ taking $h = 0.1$ given that y satisfies the Differential Equation $y' = x + 1$ with initial condition $y(0) = 1$. **(2007)**

Q11. Find the value of $y(1.2)$ using Runge-Kutta 4^{th} order method with step size $h = 0.2$ from the initial value problem $y' = xy,\ y(1) = 2$. **(2009)**

Q12. Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data

x	0.1	0.2	0.3	0.4
y	0.9975	0.9900	0.9776	0.9604

(2012)

Q13. Provide a computer algorithm to solve an O.D.E. by Euler's method to solve $\frac{dy}{dx} = f(x, y)$ in the interval $[a, b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$. **(2012)**

Q14. Use Euler's method with step size $h = 0.15$ to compute the approximate value of $y(0.6)$, correct upto 5 decimal places for

$$y' = x(y + x) - 1, \qquad y(0) = 2$$ **(2013)**

Q15. Use Runge-Kutta formula of 4^{th} order to find the value of y at $x = 0.8$, where $\frac{dy}{dx} = \sqrt{x + y}$, $y(0.4) = 0.41$. Take the step length $h = 0.2$. **(2014)**

Unit 28: Number Systems / Boolean Algebra

Q1. (i) Covert the following binary number into octal and hexadecimal system

1 0 1 1 1 0 0 1 01 0 0 1 0

(ii) Find the multiplication of the following binary numbers

1 1 0 0 1 1 and 1 0 1 ·1 **(2003)**

Q2. (i) Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011 \cdot 10)_2$ **(2005)**

(ii) Given the number 59.625 in decimal system. Write its binary equivalent.

(iii) Given the number 3898 in decimal system. Write its equivalent in system base 8.

(2006)

Q3. (i) Covert $(46655)_{10}$ into one in base 6.

(ii) $(11110 \cdot 01)_2$ into a number in the decimal system. **(2007)**

Q4. Find the values of two valued Boolean variables A, B, C, D by solving the following simultaneous equations:

$\overline{A} + AB = 0$

$AB = AC$

$AB + A\overline{C} + CD = C\overline{D}$. **(2009)**

Q5. (i) Realize the following expression by using NAND gates only $g = \left(\overline{a} + \overline{b} + c\right)\overline{d}\left(\overline{a} + e\right)f$

(ii) Find the decimal equivalent of $(357 \cdot 32)_8$. **(2009)**

Q6. If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$. **(2010)**

Q7. (i) Find the hexadecimal equivalent of the decimal $(587632)_{10}$.

(ii) Simplify the following:

(a) $a + a'b + a'b'c + a'b'c'd + ...$

(b) $x'y'z + yz + xz$

(iii) For the given set of data points $\left(x_1, f\left(x_1\right)\right), \left(x_2 f\left(x_2\right)\right), \ldots \left(x_n f\left(x_n\right)\right)$, write an algorithm to find the value of $f\left(x\right)$ by using Lagrange's interpolation formula. **(2010)**

(iv) Compute $\left(3205\right)_{10}$ to base 8. **(2011)**

Q8. Draw a flow chart for Lagrange's interpolation formula. **(2011)**

Q9. Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit

x	y	z	$f\left(x, y, z\right)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Q10. Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f\left(x, y\right)$ in the interval $\left[a, b\right]$ for n number of discrete points, where the initial value is $y\left(a\right) = \alpha$ using Euler's method. **(2012)**

Q11. Develop an algorithm for Newton-Raphson method to solve $f\left(x\right) = 0$ starting with initial iterate x_0, n be the number iterations allowed, $\in$ be the prescribed error and δ be the prescribed lower bound for $f\left(x\right)$. **(2013)**

Q12. Draw a flowchart for Simpson's 1/3rd rule. **(2014)**

Q13. For any Boolean variables x and y, show that $x + xy = x$. **(2014)**

Q14. Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z = xy + uv$. **(2014)**

Unit 29: KINEMATICS

Q1. If the velocity distribution of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{kz^2r^2}{r^5}\right)$ then determine the parameter k, such that it is a possible fluid motion. Hence find its velocity potential. **(15 Marks,2001, 2017)**

Q2. Show that the velocity potential $\phi = \frac{1}{2}a\left(x^2 + y^2 - 2z^2\right)$ satisfies the Laplace equation and determine the stream lines. **(12 Marks, 2002)**

Q3. For an incompressible homogeneous fluid at the point (x, y, z) the velocity distribution is given by $u = \frac{c^2 y}{r^2}, v = \frac{c^2 x}{r^2}, \omega = 0$, where r denotes distance from Z-axis. Show that it is a possible fluid motion and determine the surface which is orthogonal to stream lines.

(12 Marks, 2003)

Q4. If the velocity potential of a fluid is $\phi = \frac{z}{r^3}\tan^{-1}\frac{y}{z}$ then show that stream lines lie on the surfaces $x^2 + y^2 + z^2 = c\left(x^2 + y^2\right)^{\frac{2}{3}}$, c being constant. **(12 Marks, 2008)**

Q5. Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $q \to 0$ as $r \to \infty$, find the surface of constant speed.

(30 Marks, 2012)

Unit 30: EULER'S EQUATION OF MOTION

Q1. A infinite mass of fluid is acted upon by a force $\mu r^{-3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of a sphere $r = c$ in it; show that cavity will be filled up after an interval of time $\left\{\frac{2}{5\mu}\right\}^{\frac{1}{2}} c^{\frac{5}{4}}$.

(30 Marks, 2003, 2009)

Q2. State the condition under which Euler's equation of motion can be integrated. Show that $\frac{-\partial\phi}{\partial t} + \frac{q^2}{2} + v + \int \frac{dP}{P} = F(t)$, where symbols have their usual meaning. **(30 Marks, 2005)**

Q3. Liquid is contained between two parallel planes, the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated. Prove that if P be the pressure at the outer surface, the initial pressure at any point of the liquid distance r from the centre is $P\frac{\log r - \log b}{\log a - \log b}$ **(30 Marks, 2006)**

Q4. An infinite mass of homogeneous incompressible fluid is at rest subject to a uniform pressure π and contains a spherical cavity of radius a filled with gas at a pressure $m\pi$. Prove that if the inertia of the gas is neglected and Boyle's law be supposed to hold through the insuling motion, the radius of the sphere will oscillate between the values a and na, where n is determined by the equation $1 + 3m \log n - n^3 = 0$. If m be nearly equal to 1, the time of oscillation will be $2\pi\sqrt{\frac{\rho a^2}{3\pi}}$, where ρ being the density of fluid.

Q5. An infinite fluid which is contained in a spherical hollow region of radius a is initially at rest under the action of no forces. If a constant pressure P is applied at infinity, show that the time of filling up the cavity is $\pi^2 a \left(\frac{\rho}{\pi}\right)^{1/2} 2^{5/6} \left(\left|\frac{1}{3}\right.\right)^{-3}$.

Q6. Air, obeying Boyle's law is in motion in a uniform tube of small section. Prove that if ρ be the density and v velocity at a distance x from a fixed point at time t, then

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\left\{\rho\left(v^2 + k\right)\right\}.$$

Q7. A quantity of liquid occupies a length $2l$ of a straight tube of uniform small bore under the action of a force to a point in the tube varying as a distance from that point. Determine the motion and pressure.

Q8. A steady inviscid incompressible blow has a velocity field $u = fx,\ v = -fy$ and $\omega = 0$, where f is any constant. Derive an expression for the pressure field $P(x, y, z)$, if the pressure $P(0,0,0) = P_0$ and $\vec{g} = -g\hat{k}$. (12 Marks) **(2006)**

Unit 31: Potential Flow, Motion in 2 Dimensions

Q1. If complex potential $\omega = Az^2$, find velocity potential, stream lines and velocity at origin.

Q2. If $z = c\cos\omega$ and $z = c\cosh\omega$, find ϕ and ψ.

Q3. Find the lines of flow in 2 D given by $\omega = \phi + i\psi = -\frac{1}{2}n\left(x+iy\right)^2 e^{2\text{int}}$. Prove the paths of the particles of the fluid may be obtained by eliminating t from the equations

$$r\cos\left(nt+0\right)-x_0 = r\sin\left(nt+\theta\right)-y_0 = nt\left(x_0-y_0\right)$$

Q4. If a homogeneous liquid is acted on by a repulsive force from the origin, the magnitude of which at a distance r from the origin is μr per unit mass, show that it is possible for the liquid to move steadily without being constrained by any boundaries in the space between one branch of the hyperbola $x^2 - y^2 = a^2$ and the asymptotes and find the velocity potential

Q5. What arrangement of sources and sinks will give rise to the function $\omega = \log\left(z - \frac{a^2}{z}\right)$.Prove that two of the stream lines subdivide into the circle $r = a$ and the axis of Y.

Q6. In the region bounded by a fixed quadrilateral arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of the bounding radii. Show that streamlines leaving at either end at angle α with the radius is $r^2\sin\left(\alpha+\theta\right) = a^2\sin\left(\alpha-\theta\right)$.

Q7. In the 2 D motion of an infinite liquid there is a rigid boundary consisting of that part of the circle $x^2 + y^2 = a^2$, which lies in the first and fourth quadrants simple source of strength m is placed at the point $\left(f,0\right)$ where $f > a$. Prove that the speed of the fluid at the point $\left(a\cos\theta,\ a\sin\theta\right)$ of the semi-circular boundary is $\frac{4a\,mf^2 2in2\theta}{a^4 + f^4 - 2a^2 f^2\cos 2\theta}$. Find at what point of the boundary the pressure is least.

Q8. Two sources of strength m are placed at the points $(-a,0),(a,0)$ and a sink of strength $2m$ at the origin. Show that the stream lines are the curves $\left(x^2+y^2\right)=a^2\left(x^2-y^2+\lambda xy\right)$, where λ is any variable parameter. **(20 Marks, 1999)**

Q9. Between the fixed boundaries $\theta=\frac{\pi}{4}$ and $\theta=\frac{-\pi}{4}$ there is a 2 D liquid motion due to a source of strength m at the point $(r=a,\theta=0)$ and an equal sink at the point $(r=b,\theta=0)$. Show that the stream function is $-m\tan^{-1}\frac{r^4\left(a^4-b^4\right)\sin 4\theta}{r^8-\left(a^4+b^4\right)r^4\cos 4\theta+a^4b^4}$

and show that the velocity at (r,θ) is

$$\frac{4m\left(a^4-b^4\right)r^3}{\left(r^8-2a^4r^4\cos 4\theta+a^8\right)^{1/2}\left(r^3-2b^4r^4\cos 4\theta+b^8\right)^{1/4}}.$$ **(20 Marks, 1994, 1998)**

Q10. If fluid fills the region of space on positive side of the x- axis, which is a rigid boundary and if there be a source m at $(a,0)$ and an equal sink at $(0,b)$ and if the pressure on negative side be the same as at ∞. Show that the resultant pressure on the boundary is $\frac{\pi\rho m^2(a-b)^2}{2ab(a+b)}$. **(20 Marks, 1995, 2008, 2012)**

Q11. Show that the velocity potential $\phi=\frac{1}{2}\log\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$ gives a possible fluid motion. Determine the stream lines and show also that the curves of equal speed are the ovals of cassini given by $rr'=\text{constant}$. (20 Marks) **(2014)**

Unit 32: Axisymmetric Motion

Q1. Find velocity potential & stream function at any point of liquid contained between two coaxial cylinders of radii a & $b(a < b)$ when the cylinders are moved suddenly parallel to themselves in directions at right angles with velocities u & v respectively.

Q2. Consider a uniform flow U_0 in the positive x-direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points. **(2015)**

Q3. In an axisymmetric motion, show that stream function exists due to equation of continuity. Express the velocity components in terms of the stream function. Find the equation satisfied by the stream function if the flow is irrotational. **(2015)**

Q4. The Space between two concentric spherical shells of radii a, b (a < b) is filled with a liquid of density p. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y-direction, then show that the initial motion of the liquid is given by velocity potential

$$\emptyset = \frac{\left\{a^3 U\left[1+\frac{1}{2}b^3 r^{-3}\right]x - b^3 V\left[1+\frac{1}{2}a^3 r^{-3}\right]y\right\}}{(b^3 - a^3)}$$

where $r^2 = x^2 + y^2 + z^2$, the coordinates being rectangular. Evaluate the velocity at any point of the liquid. **(2016)**

Unit 33: Vortex Motion

Q1. In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Do the velocity components satisfy the Laplace equation? Justify.

(12 Marks, 2004)

Q2. In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Show that the components of velocity u, v, ω are solutions of Laplace equation

(12 Marks, 2010)

Q3. When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distance from its axis. Show that the path of each vortex is given by $\left(r^2 \sin^2 \theta - b^2\right)\left(r^2 - a^2\right)^2 = 4a^2b^2r^2 \sin^2 \theta$, θ being measured from the line through the centre perpendicular to the joint of the vortices. **(30 Marks, 2010)**

Q4. An infinite row of equidistant rectilinear vortices are at a distance "a" apart. The vortices are of the same strength k but are alternatively of opposite signs. Find the complex function that determines the velocity potential and the stream function. **(30 Marks, 2011)**

Q5. If n rectilinear vertices of the same strength k are symmetrically arranged as generators of a circular cylinder of radius "a" in an infinite liquid. Prove that vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^2}{(n-1)k}$. Find the velocity at any point of the liquid.

(30 Marks, 2013)

Q6. Prove that the necessary and sufficient condition that the vortex lines may be at right angles to the streamlines are $(u, v, \omega) = \mu\left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right)$, where μ and ϕ are functions of (x, y, z, t). **(10 Marks, 2013)**

Q7. Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$

Possess vorticity , where $\vec{q}$ (u , v , w) is the velocity in the Cartesian frame , $\vec{r}$ =(x , y , z) and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9$, z=0 ? **(10 Marks 2017)**

Unit 34: N-S Equations

Q1. Find Navier-Stokes equation for a steady laminar flow of a viscous incompressible fluid between two infinite parallel plates. **(20 Marks, 2014)**

Unit 35: Moments of Inertia

Q1. Find the M.I of a solid hemisphere about a diameter of its plane base. **(1999)**

Q2. Find the M.I. of an elliptic area about a line CP inclined at θ to the major axis and about a tangent parallel to CP, where C is the centre of ellipse. **(2000)**

Q3. Determine the M.I of a uniform hemisphere about its axis of symmetry and about an axis perpendicular to the axis of symmetry and through centre of its base **(2001)**

Q4. Find the M.I of a circular wire about (i) a diameter and (ii) a line through the centre and perpendicular to its plane. **(2002)**

Q5. A solid body of density ρ is in the shape of the solid formed by the revolution of the cardiod $r = a(1+\cos\theta)$ about the initial line. Show that its M.I about the straight line through the pole and perpendicular to the initial line is $\frac{352}{105}\pi\rho a^2$. **(2003)**

Q6. The flat surface of a hemisphere of radius r is cemented to one flat surface of a cylinder of the same radius and of the same material. If the length of the cylinder be 'l' and the total mass be 'm', show that the M.I of the combination about the axis of cylinder is $mr^2 \frac{\left(\frac{l}{2}+\frac{4}{15}r\right)}{\left(l+\frac{2r}{3}\right)}$ **(2009)**

Q7. Let 'a' be the radius of the base of a right circular cone of height h and mass M. Find the M.I of that right circular cone about a line through the vertex perpendicular to the axis. **(2011)**

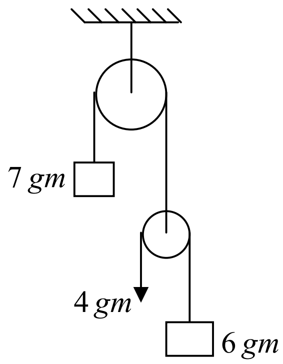
Q8. A pendulum consists of a rod of length $2a$ and mass M to one end of which a spherical bob of radius $\frac{a}{3}$ and mass 15 m is attached. Find the M.I of the pendulum:

(i) about an axis through the other end of the rod and at right angles to the rod

(ii) about a parallel axis through the centre of mass of the pendulum. [Given the centre of mass of the pendulum is $\frac{a}{12}$ above the centre of sphere **(15 + 15 = 30 Marks, 2012)**

Q9. Four solid sphere A, B, C and D each of the mass m and radius "a", are placed with their centres on the four corners of a square of side "b". Calculate the M.I of the system about a diagonal of the square **(10 Marks, 2013)**

Unit 36: Lagrange's Equation of Motion



Q1. A pulley system is given as shown in diagram. Discuss the motion of the system, using Lagrange's method the pulley wheels have negligible masses and moment of inertia and there wheels are frictionless. **(1997)**

Q2. Using Lagrange's equations obtain the differential equation of planetary motion **(1997)**

Q3. Using Lagrange's equation, obtain the differential equation of motion of a free particle in spherical polar coordinates. **(1998)**

Q4. Two particles in a plane are connected by a rod of length and are constrained to move in such a manner the velocity of the middle of the rod is in the direction of rod. Write down the equation of constraints. Is the system holonomic or non-holonomic. Give reasons for your answer. **(1998)**

Q5. A particle of mass m moves in space with Lagrengian $L = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - V + \dot{x}A + \dot{y}B + \dot{z}C$, where V, A are given functions of x, y, z. Show that the equation of motion are $m\ddot{x} = -\frac{\partial V}{\partial x} + \dot{y}\left[\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right] - \dot{z}\left[\frac{\partial A}{\partial z} - \frac{\partial C}{\partial x}\right]$ and two similar equations for y and z. Find also the Hamiltonian H in terms of generalized momenta. **(1997)**

Q6. Find the equation of motion for a particle of mass m which is constrained to move on the surface of a cone of Semi-Vertical angle α and which is subjected to a gravitational force. **(2001)**

Q7. A uniform rod of mass $3m$ and length $2l$ has its middle point fixed and a mass m is attached to one of its extremely. The rod, when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity $\sqrt{\frac{28}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is **(2008)**

Q8. Obtain the equations governing the motion of a spherical pendulum. **(2012)**

Unit 37: Hamilton's Equation of Motion

Q1. Derive the Hamitlon equations of motion from the principle of least action and obtain the same for a particle of mass m moving in a force field of potential V. Write these equations in spherical coordinates (r,θ,ϕ). **(2004)**

Q2. A particle of mass m is constrained to move on the surface of a cylinder. The particle is subjected to a force directed towards the origin and proportional to the distance of the particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion. **(2006)**

Q3. A point mass m is placed on a frictionless plane that is tangent to the earth's surface. Determine Hamilton's equations by taking x or θ as the generalized coordinate. **(2007)**

Q4. A sphere of radius 'a' and mass M rolls down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **(2010)**

Q5. Find the equation of motion of a compound pendulum using Hamilton's equations. **(2014)**

Unit 38: Equation of Motion in 2 D / De' Alembert's Principle

Q1. A uniform rod OA of length $2a$ free to turn about its end O revolves with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ. Show that the value of α is either O or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$. **(1996)**

Q2. A planet of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizontal and a man of mass M. Starting from the upper end walks down the plank. So that it does not move. Show that he gets to other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$, where a is length of the plank **(2000, 2005)**

Q3. A perfectly rough sphere of mass m and radius b rests on the lowest point of a fixed spherical cavity of radius 'a'. To the highest point of a movable sphere is attached a particle of mass 'M' and the system is disturbed. Show that the oscillations are the same as those of simple pendulum of length $(a-b)\ 4m'+7/5\,m/m+m'\left(2-\frac{a}{b}\right)$.

Framing of P.D.E. & Linear P.D.E. of Order-One

Assignment - 1

Q1. Solve $\left(x^3+3xy^2\right)p+\left(y^3+3x^2y\right)q=2\left(x^2+y^2\right)z$. **(1993)**

Q2. Find the integral surface of the P.D.E. $\left(x-y\right)p+\left(y-x-z\right)q=1z$ through the circle $z=1,\ x^2+y^2=1$. **(1993)**

Q3. Find the integral surface of $x^2p+y^2q+z^2=0$, which passes through the hyperbola $xy=x+y, z=1$. **(1994)**

Q4. Obtain a complete solution of $pq=x^m y^n z^l$. **(1994, 2000)**

Q5. In the context of a P.D.E. of the first order in three independent variables, define and illustrate the terms:

(i) the complete integral (ii) the singular integral **(1995)**

Q6. Find the general integral of $\left(y+z+\omega\right)\frac{\partial\omega}{\partial x}+\left(z+x+\omega\right)\frac{\partial\omega}{\partial y}+\left(x+y+\omega\right)\frac{\partial\omega}{\partial z}=x+y+z$.

(1995)

Q7. Find the integral surface of the equation $\left(x-y\right)y^2p+\left(y-x\right)x^2q=\left(x^2+y^2\right)z$ passing through the curve $xz=a^3,\ y=0$. **(1996)**

Q8. Form differential equation by eliminating f and g from $z=f\left(x^2-y\right)+g\left(x^2+y\right)$.

(1996)

Q9. Find the D.E. by eliminating a and b from $z=\left(x^2+a\right)\left(y^2+b\right)$. **(1997)**

Q10. Find the equation of surfaces satisfying $4yzp+q+2y=0$ and passing through $y^2+z^2=1,\ x+z=2$. **(1997)**

Q11. Solve: $\left(y+z\right)p+\left(z+x\right)q=x+y$. **(1997)**

Q12. Form the D.E. by eliminating $a, b\ \&\ c$ from $z=a\left(x+y\right)+b\left(x-y\right)+abt+c$. **(1998)**

Q13. Solve: $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\frac{z\partial u}{\partial z}=xyz$. **(1998)**

Q14. Find the internal surface of the linear P.D.E. $x\left(y^2+z\right)\frac{\partial z}{\partial x}-y\left(x^2+z\right)\frac{\partial z}{\partial y}=y\left(x^2+y^2\right)z$ through the straight line $x+y=0, z=1$. **(1998)**

Q15. Verify that the D.E. $\left(y^2+yz\right)dx+\left(xz+z^2\right)dy+\left(cy^2-xy\right)dz=0$ is integrable and find its primitive.

Q16. Find the surface which intersects the surfaces of the system $z\left(x+y\right)=\left(3z+1\right), C=$ a constant, orthogonally and which passes through the circle $x^2+y^2=1, z=1$

Q17. Find the characteristics of the equation $pq=z$ and determine the internal surface which passes through the parabolic $x=0, y^2=z$ **(1998)**

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