CIVIL SERVICES EXAMINATION (MAINS)

MATHEMATICS PAPER I: Analytic Geometry

Unit 16: (Plane)

Q1. Find the equation of the plane parallel to the plane $ax+by+cz=0$ and passing through the point (α,β,γ).

Q2. Find the equation of a plane- xy plane and passing through the points $(1,0,5),(0,3,1)$.

Q3. Find the equation of the plane passing through the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point $(1,1,1)$.

Q4. Find the equation of the plane through the points $(2,2,1)$ and $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

Q5. the plane $lx+my=0$ is rotated about its line of intersection with the plane $z=0$ through an angle α . Prove that the equation of plane in its new position is $lx+my\pm z\sqrt{l^2+m^2}\tan\alpha=0$.

Q6. A variable plane is at a constant distance p from the origin and meets the axes, which are rectangular in A,B,C. Prove that the locus of the point of intersection of the planes through A,B,C parallel to coordinate planes is

$$x^{-2}+y^{-2}+z^{-2}=p^{-2}$$

Q7. Prove that $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ represents a pair of planes.

Q8. From a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the coordinate axes is A',B',C'. Prove that the area of the ΔABC is $\frac{r^5}{2x'y'z'}$ where $r=0P$

Q9. Two system of rectangular axes have the same origin. A plane cuts off intercepts a,b,c,a',b',c' from the axes respectively. Prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}$.

Q10. A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Show that the locus of the centroid of tetrahedron O,A,B,C is $x^{-2}+y^{-2}+z^{-2}=16p^{-2}$.

Q11. A point P moves on a fixed plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. The plane through P perpendicular to OP meets the axis in A,B,C. The plane through A,B,C parallel to coordinate axes intersect in θ. Show that the locus of θ

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$$

Q12. The plane $x-2y+3z=0$ is rotated through a right angle about its line of intersection with the plane $2x+3y-4z-5=0$. Find the equation of the plane in its new position.

(2008)

Unit 17: Straight Line

Q1. A line with direction ratios 2, 7, -5 is drawn to intersect the lines

$$\frac{x}{3}=\frac{y-1}{2}=\frac{z-2}{4} \text{ and } \frac{x-11}{3}=\frac{y-5}{-1}=\frac{z}{1}$$

Find the coordinates of the points of intersection and the length intercepted on it. **(2007)**

Q2. Find the locus of the point which moves so that its distance from the plane $x+y-z=1$ is twice its distance from the line $x=-y=z$. **(2007)**

Q3. A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ to meet two fixed lines $y=mx, z=c$ and $y=-mx, z=-c$. Find the locus of the line. **(2008)**

Q4. Find the equations of the straight line through the point $(3,1,2)$ to intersect the straight line $x+4=y+1=2(z-2)$ and parallel to the plane $4x+5z+y=0$. **(2011)**

Q5. Find the equation of the plane which passes through the points $(0,1,1)$ and $(2,0,-1)$ and parallel to the line joining the points $(-1,1,-2)$, $(3,-2,4)$. Find also the distance between the plane and line. **(2013)**

Q6. Find the image of the point $(1,2,3)$ in the plane $2x-3y+6z+35=0$. **(2007)**

Q7. Find the distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$.

Q8. Find the S.D. between the lines and its equation $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$

Q9. Find the shortest distance between the lines and its equation $3x-9y+5z=0=x+y-z$ and $6x+8y+3z-13=0=x+2y+z-3$.

Q10. Show that the equation to the plane containing the line $\frac{y}{6}+\frac{z}{c}=1;\ x=0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1,\ y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if 2d is the S.D. prove that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$.

Q11. Prove that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0, z+x=0, x+y=0, x+y+z=a$ is $\frac{2a}{\sqrt{6}}$ and three lines of S.D. intersect at the point $x=y=z=-a$.

Q12. If the axes are rectangular the S.D. between the line $y=az+b, z=ax+\beta; y=a'z+b', z=\alpha'x+\beta'$ is

$$\frac{(\alpha-\alpha')(b-b')-(\alpha'\beta-\alpha\beta')(a-a')}{\left[\alpha^2\alpha'^2(a-a')^2+(\alpha-\alpha')^2+(a'\alpha'-a\alpha)^2\right]^{1/2}}$$

Q13. Prove that the lines $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-4}{5}$ and $2x-3y+z=0=x+y-2z+20$ are coplanar. Find also their point of intersection.

Q14. Find the equation of the plane through the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{l}$ and $\frac{x}{n}=\frac{y}{l}=\frac{z}{m}$.

Q15. Prove that all lines which intersect the lines $y=mx, z=c; y=-mx, z=-c;$ and the x-axis lie on the surface $mxz=cy$.

Q16. Prove that locus of a variable line which intersects the three gives lines $y=mx, z=c; y=-mx, z=-c, y=z, mx=-c$ is the surface ___________(Please Check)

Unit 18: SPHERE

Q1. Find the equation of sphere passing through the points $(0,0,0)(0,1,,-1)(-1,2,0)(1,2,3)$, coordinates of centre and its radius.

Q2. Find the equation to the sphere through the circle $x^2+y^2+z^2=9,\ 2x+3y+4z=5$ and the origin.

Q3. Find the equation of Tangent planes to the sphere $x^2+y^2+z^2-zx+4y-6z+13=0$ which are parallel to the plane $x-y+z=0$

Q4. Find the equation of the sphere which passes through the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touch the plane $4x+3y=15$

Q5. A sphere of radius k passes through the origin and meets the axes at A,B,C. Prove that the centroid of the triangle ABC lies on the sphere $a\left(x^2+y^2+z^2\right)=4k^2$

Q6. A variable plane is parallel to the given plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes in A,B,c

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+xz\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$$

Q7. A plane through a fixed point (a,b,c) cuts the axes in A,B,C. Show that the focus of the centre of sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$, 0 being the origin

Q8. Find out the equation of sphere passing through the origin and meeting the axis of x,y,z respectively at A,B,C.

Q9. Prove that the circle $x^2+y^2+z^2-2x+3y+4z-5=0, 5y+6z+1=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on the same sphere. Also find the value of a for which $x+y+z=\frac{a}{\sqrt{3}}$ touches the sphere.

Q10. Find the equation of a sphere which touches the sphere $x^2+y^2+z^2+2x-6y+1=0$ at $(1,2,-2)$ and passes through the origin.

Q11. Find the equation of a sphere inscribed in the tetrahedron whose faces are $x=0, y=0, z=0, zx-6y+3z+6=0$

Q12. Prove that the centres of spheres which touch the lines $y = mx \quad z = c; \quad y = -mx, \quad z = -c$ lie upon the conicoid $mxy + cz\left(1+m^2\right) = 0$

Q13. If any tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on the coordinate axes, prove that $a^2 + b^2 + c^{-2} = r^{-2}$

Q14. Shoe that the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ intersect orthogonally. Find their planes of intersection.

Q15. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$

Q16. A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Prove that the focus of the foot of the perpendicular from O to the line ABC is given by $\left(x^2 + y^2 + z^2\right)^2 \left(x^2 + y^{-2} + z^{-2}\right) = 4r^2$

Unit 19: CONE & CYLINDER

Q1. Find the equation of right circular cylinder whose axis is $x = 2y = -z$ and radius 4. Prove that area of cross-section of this cylinder by the plane $z = 0$ is 24π

Q2. Find the equation of the right circular cylinder which passes through circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$

Q3. Find the equation to the cylinder whose generators are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and which envelops the surface $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Q4. Find the equation of the cylinder whose generator are parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and passes through the curve $x^2 + y^2 = 16, z = 0$

Q5. Find the equation of the cylinder which intersects the curve $ax^2 + by^2 + cz^2 = 1$, $lx + my + nz = p$ and whose generators are parallel to z - axis.

Q6. Show that the equation to the right circular cylinder described on the circle through three points $(1,0,0)(0,1,0)$ and $(0,0,1)$ as girding curve is $x^2 + y^2 + z^2 - yz - zx - xy = 1$

Q7. Show that $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{\omega^2}{c} = d$

Q8. Find the equation of a cone whose vertex is the point (α, β, γ) and whos generating lines pass through the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$

Q9. Find out the vertex of the cone $2y^2 - 8yz - 4xz - 8xy + 6x - 4y - 2z + 5 = 0$

Q10. Find the equation of the cone whose vertex is $(1,2,3)$ and griding curve is the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 1$

Q11. Find the equation of the cone with vertex at $(0,0,0)$ and which passes through the curve $ax^2 + by^2 + cz^2 - 1 = 0 = \alpha x^2 + \beta y^2 - 2z$

Q12. The section of a cone whose vertex is p and guiding curve the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane $x=0$ is a rectangular Hyperbola. Show that focus of p is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$

Q13. find the equations to the lines in which the plane $2x+y-z=0$ cuts the cone $4x^2-y^2+3z^2=0$

Q14. Prove that the plane $ax+by+cz=0$ cuts the cone $xy+yz+xz=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

Q15. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represent one of a set of three mutually perpendicular generators of the cone $5yz-8xz-3xy=0$ find the equation of the other two

Q16. Prove that cones $ax^2+by^2+cz^2=0$ and $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=0$ are reciprocal

Q17. Prove that the angle between the lines given by $x+y+z=0, ayz+bxz+cxy=0$ is $\frac{\pi}{2}$ if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

Q18. Prove that the equation $\sqrt{fx}+\sqrt{gy}+\sqrt{hz}=0$ represent a cone which touches the coordinate planes and that the equation of its reciprocal cone is $fyz+gzx+hxy=0$

Q19. Find the laws of the vertices of enveloping cones by the plane $z=0$ are circles.

Unit 20: Conicoids

Q1. Prove that the lines of intersection of pairs of tangent planes to $ax^2+by^2+cz^2=0$ which touch along perpendicular generators lie on the cone

$$a^2(b+c)x^2+b^2(c+a)y^2+c^2(a+b)z^2=0.$$ **(2004)**

Q2. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ to them through the origin generate the cone

$$(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$$ **(2004)**

Q3. Obtain the equation of right circular cylinder on the circle through the points $(a,0,0)(a,b,0)$ and $(0,0,c)$ as the grinding curve. **(2005)**

Q4. Show that the plane $2x-y+2z=0$ cuts the cone $xy+yz+zx=0$ in perpendicular lines. **(2007)**

Q5. If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represent one of a set of three immutably + generators of the cone $5yz-8xz-3xy=0$ find the equation of other two. **(2008)**

Q6. Prove that the normals from the point (α,β,γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lies on the cone $\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$. **(2009)**

Q7. Find the vertices of the show quadrilateral formed by the four generator of the Hyperboloid and $(14,2,-2)\frac{x^2}{4}+y^2-z^2=49$ passing through $(10,5,1)$. **(2010)**

Q8. Three points P,Q,R are taken on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ so that the lines joining P,Q,R to the origin are mutually perpendicular. Prove that the plane P,Q,R touches a fixed sphere. **(2011)**

Q9. Show that the generators through any one of the ends of an equicojugate diameter of the principal elliptic section of the Hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of 60^0 if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. **(2011)**

Q10. Show that the locus of a point from which the these mutually perpendicular tangent lines can be drawn to paraboloid $x^2 + y^2 + 2z = 0x^2 + y^2 + 4z = 1$. **(2012)**

Unit 21: 2nd Degree Equation, Reduction to Canonical Form

Q1. Reduce the equation:

$11x^2+10y^2+6z^2-8yz+4zx-12xy+7zx-7zy+36z+150=0$ to the standard form and give the nature of the surface. Also find the equations of its axes.

Q2. Prove that the equation $x^2+y^2+z^2+yz+zx+xy+3x+y+4z+4=0$ represents an ellipsoid the squares of whose semi axes are $2,2,\frac{1}{2}$. Show that its principal axis is given by $x+1=y-1=z+2$.

Q3. Show that the equation $2y^2+4zx+2x-4y+6z+5=0$ represents a right circular cone. Show also that the semi-vertical angle of this cone is $\frac{\pi}{4}$ and that its axis is given by $x+z+2=0,\ y=1$.

Q4. Show that $2x^2+2y^2+z^2+2yz-2xz-4xy+x+y=0$

Q5. Reduce $3z^2-6yz-6zx-7x-5y-6z+3=0$ to standard for and find the nature of the surface represented by this equation.

Q6. Prove that $5x^2+5y^2+8z^2+8yz+8zx-2xy+12x-12y+6=0$ represents a cylinder whose cross-section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$ and find the equations to its axis.

Q7. Reduce the surface $36x^2+4y^2+z^2-4yz-12zx+24xy+4x+16y-26z-3=0$ to the standard form and find the lotus rectum of a normal section.

Most general equation of 2nd degree in 3 coordinates:

$F(x,y,z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy+24x+2vy+2wz+d=0$

It can be reduced to any one of the below mentioned forms by transformation of axes:

$$\lambda_1x^2+\lambda_2y^2+\lambda_3z^2=\mu \qquad (1)$$

$$\lambda_1x^2+\lambda_2y^2=2\mu z \qquad (2)$$

By giving different values to $\lambda_1,\lambda_2,\lambda_3\ \&\ \mu$ from (1) can be reduced to

(i) $Ax^2+By^2+Cz^2=1$ — Ellipsoid

(ii) $A(x^2+y^2)+Cz^2=1$ — Ellipsoid of revolution

(iii) $A\left(x^2+y^2+z^2\right)=1$	Sphere
(iv) $Ax^2+By^2-Cz^2=1$	Hyperboloid of one sheet
(v) $Ax^2-By^2-Cz^2=1$	Hyperboloid of two sheet
(vi) $A\left(x^2-z^2\right)+By^2=1$	Hyperboloid of revolution
(vii) $Ax^2+By^2+Cz^2=0$	Cone
(viii) $Ax^2+By^2=1$	Elliptic cylinder
(ix) $Ax^2-By^2=1$	Hyperbolic cylinder
(x) $Ax^2-By^2=0$	Pair of intersecting planes
(xi) $Ax^2=1$ or $By^2=1$ or $Cz^2=1$	Pair of parallel planes

Homogenous part $f\left(x,y,z\right)=ax^2+by^2+\left(z^2+2fyz+2gzx+2hxy\right)$.

CIVIL SERVICES EXAMINATION (MAINS)

MATHEMATICS PAPER I: O.D.E

Unit 22: Diffrential equations of First order and First Degree

Q1. Solve $x\frac{dy}{dx}+y\log y=xye^{x}$ **(2003)**

Q2. Solve $\frac{dy}{dx}+y\cos x=\frac{1}{2}\sin 2x$ **(2004)**

Q3. Solve $y\left(xy+2x^{2}y^{2}\right)dx+x\left(xy-x^{2}y^{2}\right)dy=0$ **(2004)**

Q4. Solve $xy\frac{dy}{dx}=\sqrt{x^{2}+y^{2}-x^{2}y^{2}-1}$ **(2005)**

Q5. Solve the D.E $\left(xy^{2}+e^{-\frac{1}{x^{3}}}\right)dx-x^{2}y\,dy=0$ **(2006)**

Q6. Solve $\left(1+y^{2}\right)+\left(x-e^{-\tan^{-1}y}\right)\frac{dy}{dx}=0$ **(2006)**

Q7. Solve the D.E

$$\cos 3x\frac{dy}{dx}-3y\sin 3x=\frac{1}{6}\sin 6x+\sin^{2}3x \qquad 0<x<\frac{\pi}{2}$$ **(2007)**

Q8. Solve $\frac{dy}{y}+xy^{2}dx=-4xdx$ **(2007)**

Q9. Solve $ydx+\left(x+x^{3}y^{2}\right)dy=0$ **(2008)**

Q10. Solve $\frac{dy}{dx}=\frac{y^{2}\left(x-y\right)}{3xy^{2}-x^{2}y-4y^{3}}, \qquad y\left(0\right)=1$ **(2009)**

Q11. Show that D.E.

$\left(3y^{2}-x\right)+2y\left(y^{2}-3\right)y'=0$ admits an integrating factor which is a function of $\left(x+y^{2}\right)$.

Hence solve the equation. **(2010)**

Q12. Verify that

$$\frac{1}{2}\left(Mx+Ny\right)d\left(\ln xy\right)+\frac{1}{2}\left(Mx-Ny\right)d\ln\left(\frac{x}{y}\right)=Mdx+Ndy$$

Hence show that:

(i) If the D.E. $Mdx+Ndy=0$ is homogeneous, then $Mx+Ny$ is an I.F. unless $Mx+Ny=0$

(ii) If the D.E $Mdx+Ndy=0$ is not exact but is of the form

$f_1(x,y)ydx+f_2(x,y)xdy=0$, then $\frac{1}{Mx-Ny}$ is an I.F. unless $Mx-Ny=0$. **(2010)**

Q13. Solve $\frac{dy}{dx}=(4x+y+1)^2$

Q14. Solve $\frac{dy}{dx}=\frac{2xy\,e^{\left(\frac{x}{y}\right)^2}}{y^2\left(1+e^{\left(\frac{x}{y}\right)^2}\right)+2x^2e^{\left(\frac{x}{y}\right)^2}}$ **(2012)**

Q15. Show that the D.E.

$(2xy\log y)dx+\left(x^2+y^2\sqrt{y^2+1}\right)dy=0$ is not exact. Find an integrating factor and hence the solution of the equation. **(2012)**

Q16. Solve: $\frac{dy}{dx}=\cos(x+y)+\sin(x+y)$ **(2013)**

Q17. Solve $(5x^3+12x^2+6x)dx+6xy\,dy=0$ **(2013)**

Unit 23: D.E. of 1st order and Higher Degree

Q1. Solve the D.E $\left(px^2+y^2\right)\left(px+y\right)=\left(p+1\right)^2$ by reducing to Clairut's form using suitable substitutions. **(2003)**

Q2. Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal **(2003)**

Q3. Reduce the equation to Clairut's equation and solve it:

$$\left(px-y\right)\left(py+x\right)=2p \text{ where } p=\frac{dy}{dx}$$ **(2004)**

Q4. Solve the D.E by reducing to it to Clairut's from by using suitable substitution

$$\left(x^2+y^2\right)\left(1+p\right)^2-2\left(x+y\right)\left(1+p\right)\left(-x+yp\right)+\left(x+yp\right)^2=0$$ **(2005)**

Q5. Find the orthogonal trajectory of a system of coaxial circles $x^2+y^2+2gx+c=0$ where g is the parameter. **(2005)**

Q6. Solve $x^2p^2+yp\left(2x+y\right)+y^2=0$, using the substitution $y=u$ and $xy=v$ and find its singular solution where $p=\frac{dy}{dx}$ **(2006)**

Q7. Find the family of curves whose tangents form an angle $\frac{\pi}{4}$ with the hyperbolas $xy=c, c>0$. **(2006)**

Q8. Solve the equation $y-2xp+yp^2=0$ where $p=\frac{dy}{dx}$. **(2008)**

Q9. Determine the orthogonal trajectory of a finally of curves represented by the polar equation $r=a\left(1-\cos\theta\right)$. **(2011)**

Q10. Obtain Chairut's form of the D.E.

$\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+x\right)=a^2\frac{dy}{dx}$. Also find its general solution. **(2011)**

Q11. Find the orthogonal trajectories of the family of curves $x^2+y^2=ax$. **(2012)**

Q12. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n=a\sin n\theta$ **(2013)**

Unit 24: D.E. of 2nd order with constant coefficients

Q1. Solve $\left(D^5 - D\right)y = 4\left(e^x + \cos x + x^3\right)$ where $D = \frac{d}{dx}$. **(2003)**

Q2. Solve $\left(D^4 - 4D^2 - 5\right)y = e^x\left(x + \cos x\right)$ **(2004)**

Q3. Solve $\left(D^2 - 2D + 2\right)y = e^x \tan x$ **(2006)**

Q4. Solve $\left(D^3 - 6D^2 + 12D - 8\right)y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right)$ **(2007)**

Q5. Obtain the general solution: $y'' - 2y' + 2y = x + e^x \cos x$ **(2011)**

Q6. Find the general solution of the equation $y''' - y'' = 12x^2 + 6x$ **(2012)**

Unit 25: 2nd Order D.E. with Variable Coefficients

Q1. Solve $(1+x)^2 y'' + (1+x)y' + y = \sin\{2\log(1+x)\}$. **(2003)**

Q2. Solve the D.E. by Variation of parameters: $x^2 y'' - 4xy' + 6y = x^4 \sec^2 x$ **(2003)**

Q3. Solve $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$ **(2004)**

Q4. Solve: $(1-x^2)\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - (1+x^2)y = x$ **(2004)**

Q5. Solve: $\left[(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)\right]y = \frac{1}{x+1}$. **(2005)**

Q6. Solve the D.E.: $(\sin x - x\cos x)y'' - x\sin xy' + y\sin x = 0$, given that $y = \sin x$ is a solution of this equation. **(2005)**

Q7. Solve the D.E. by variation of parameters: $x^2 y'' - 2xy' + 2y = x\log x, \quad x > 0$ **(2005)**

Q8. Solve: $x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{y}{x} = 10\left(1 + \frac{1}{x^2}\right)$ **(2006)**

Q9. Solve: $2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = x^3$ **(2007)**

Q10. Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x$ **(2007)**

Q11. Use the method of variation of parameters to find the general solution of $x^2 y'' - 4xy' + 6y = -x^4 \sin x$. **(2008)**

Q12. Solve the D.E.: $x^3 y'' - 3x^2 y' + xy = \sin(\ln x) + 1$. **(2008)**

Q13. Solve by the method of Variation of parameters: $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **(2011)**

Q14. Solve the D.E.: $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$ **(2012)**

Q15. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + a^2 y = \sec ax$. **(2013)**

Q16. Find the general solution of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \ln x \sin(\ln x)$ **(2013)**

Q17. Solve the D.E.: $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\ln x)$ **(2014)**

Q18. Solve the following D.E.: $x\frac{d^2 y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$, when e^x is a solution to its corresponding homogenous D.E. **(2014)**

Q19. Solve by the method of variations of parameters $\frac{dy}{dx} - 5y = \sin x$. **(2014)**

Unit 26: Laplace Transforms

Q1. Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$ with $y(0) = 1, \quad y'(0) = -1$ **(2008)**

Q2. Find the inverse Laplace transform of $F(s) = \ln\left(\frac{s+1}{s+5}\right)$. **(2009)**

Q3. Find the D.E. of the family of circles in the xy - plane passing through $(-1,1)$ and $(1,1)$. **(2009)**

Q4. Use Laplace transform to solve: $\frac{d^2y}{dx^2} - 2\frac{dx}{dt} + x = e^t, \quad x(0) = 2$ and $\left.\frac{dx}{dt}\right|_{t=0} = -1$. **(2011)**

Q5. Using Laplace transforms, solve the initial value problem

$$y'' + 2y' + y = e^{-t}, \qquad y(0) = -1, y'(0) = 1$$

(2012)

Q6. Using Laplace transform method, solve: $(D^2 + n^2)x = a\sin(nt + \alpha)$ with condition at $x = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$. **(2013)**

Q7. Solve the initial value problem using Laplace transform:

$$\frac{d^2y}{dt^2} + y = 8e^{-2t}\sin t, \quad y(0) = 0, y'(0) = 0.$$

(2014)

Mathematical Paper 1: Section B / Vector Analysis

Unit 27: Scalar & Vector Fields, Triple Products, Differentiation of vector

Q1. If $\vec{a}\times\vec{r}=\vec{b}+\lambda\vec{a}$ and $\vec{a}.\vec{r}=3$ where $\vec{a}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=-\hat{i}-2\hat{j}+\hat{k}$, then find $\vec{r}$ and λ

Q2. If $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\vec{b}}{2}$. Find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c},\vec{b}$ and $\vec{c}$ being non parallel.

Q3. If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the vertices A,B,C of a triangle, show that vector area of the Δ is $\frac{1}{2}.(\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b})$

Q4. Prove that $[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}]=2(a\,b\,c)$

Q5. Show that the four points whose position vectors are $3\hat{i}-2\hat{j}+4\hat{k},6\hat{i}+3\hat{j}+\hat{k},5\hat{i}+7\hat{j}+3\hat{k}$ and $2\hat{i}+2\hat{j}+6\hat{k}$ are coplanar

Q6. Prove the identity

(i) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=\begin{vmatrix}\vec{a}.\vec{c} & \vec{a}.\vec{d}\\ \vec{b}.\vec{c} & \vec{b}.\vec{d}\end{vmatrix}$

(ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\vec{b}\vec{d}]\vec{c}-[\vec{a}\vec{b}\vec{c}]\vec{a}$

Q7. If $\frac{d\vec{u}}{dt}=\vec{\omega}\times\vec{u},\frac{d\vec{v}}{dt}=\vec{\omega}\times\vec{v}$, show that $\frac{d}{dt}(\vec{u}\times\vec{v})=\vec{\omega}\times(\vec{u}\times\vec{v})$

Q8. If $\vec{R}$ be a unit vector in the direction of $\vec{r}$, prove that $R\times\frac{d\vec{R}}{dt}=\frac{1}{r^2}\vec{r}\times\frac{d\vec{r}}{dt}$

Q9. If $\vec{r}(t)=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$, prove that $\int_1^2\left(\vec{r}\times\frac{d^2r}{dt^2}\right)dt=-14\hat{i}+75\hat{j}-15\hat{k}$

Q10. Let $\vec{R}$ be the unit vector along the vector $\vec{r}(t)$. Show that $\vec{R}\times\frac{d\vec{R}}{dt}=\frac{\vec{r}}{r^2}\times\frac{d\vec{r}}{dt}$ **(2002)**

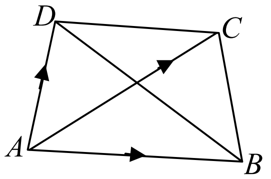
Q11. Show that if $\vec{a}',\vec{b}'\,\&\,\vec{c}'$ are the reciprocals to the non coplanar vector $\vec{a},\vec{b},\vec{c}$, then any vector $\vec{r}$ may be written as

$$\vec{r}=(\vec{r}\cdot\vec{a}')\vec{a}+(\vec{r}\cdot\vec{b}')\vec{b}+(\vec{r}\cdot\vec{c}')\vec{c}$$ **(2003)**

Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$ where t is the time.

Q12. Find the components of its acceleration along the radial and transverse directions. **(2003)**

Q13. Show that the volume of tetrahedron ABCD is $\frac{1}{6}\left(\vec{AB}\times\vec{AC}\right)\cdot\vec{AD}$. Hence find the volume of the tetrahedron with vertices $(2,2,2),(2,0,0),(0,2,0)\&(0,0,2)$.



(2005)

Q14. If $\vec{A}=2\hat{i}+\hat{k},\vec{B}=\hat{i}+\hat{j}+\hat{k},\vec{C}=4\hat{i}-3\hat{j}-7\hat{k}$ determine a vector $\vec{R}$ satisfying $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$ **(2006)**

Q15. Show that $\vec{F}=\left(2xy+z^3\right)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential $\vec{F}$ and work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$.

(2008)

Q16. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1,\ z=0$ under the field of force given by

$$\vec{F}=(2x-y+z)\hat{i}+(x+y-z)\hat{j}+(3x-2y+4z)\hat{k}.$$ **(2009)**

Q17. Show that the vector field defined by the vector function $\vec{V}=xyz\left(yz\hat{i}+xz\hat{j}+xy\hat{k}\right)$ is conservative. **(2010)**

Q18. For any vectors $\vec{a}\,\&\,\vec{b}$ given respectively by $\vec{a}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$ and $\vec{b}=\sin t\hat{i}-\cos t\hat{j}$ determine (i) $\frac{d}{dt}\left(\vec{a}\vec{b}\right)$ and (ii) $\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$. **(2011)**

Q19. Examine within the vectors $\vec{\nabla}u,\vec{\nabla}v\,\&\,\vec{\nabla}w$ are coplanar, where u,v,w are the scalar function whether defined by

$$u=x+y+z$$

$$v=x^2+y^2+z^2$$

$w = yz + xz + xy$ **(2011)**

Q20. if $\vec{A} = x^2 yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$

$\vec{B} = 2z\hat{i} - y\hat{j} - x^2\hat{k}$

Find the value of $\frac{\partial^2}{\partial xy}\left(\vec{A}\times\vec{B}\right)$ at $(1,0,-2)$. **(2012)**

CIVIL SERVICES EXAMINATION (MAINS)

Unit 28: Gradient, Divergence and Curl

Q1. If $\vec{a}\,\&\,\vec{b}$ are constant vectors, then show that:

(i) $\vec{\nabla}\cdot\{\vec{x}\times(\vec{a}\times\vec{x})\}=-2\vec{x}\cdot\vec{a}$

(ii) $\vec{\nabla}\cdot\{(\vec{a}\times\vec{x})\times(\vec{b}\times\vec{x})\}=2\vec{a}\cdot(\vec{b}\times\vec{x})-2\vec{b}\cdot(\vec{a}\times\vec{x})$ **(1992)**

Q2. Prove that the angular velocity of rotation at any point is equal to one half of the curl of the velocity vector $\vec{V}$. **(1993)**

Q3. Show that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n=-3$. **(1994, 2006)**

Q4. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$, show that:

(i) $\vec{r}\times\text{grad}\, f(r)=0$

(ii) $\vec{\nabla}\cdot(r^n\vec{r})=(n+3)r^n$ **(1996)**

Q5. If $\vec{r}_1$ and $\vec{r}_2$ are the vectors joining the fixed points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ respectively to a variable point $P(x,y,z)$, then find the values of Grad $(\vec{r}_1\cdot\vec{r}_2)$ and $\vec{r}_1\times\vec{r}_2$. **(1998)**

Q6. Evaluate $\vec{\nabla}\times\vec{F}$ for $\vec{F}=\vec{\nabla}(x^3+y^3+z^3-3xyz)$. **(1999)**

Q7. In what direction from the point $(-1,1,1)$ is the directional derivative of $f=x^2yz^3$ is maximum? Compute the magnitude. **(2000)**

Q8. Show that the vector field defined by $\vec{F}=2xyz^3\hat{i}+x^2z^3\hat{j}+3x^2yz^2\hat{k}$ is irrotational. Find also the scalar U such that $\vec{F}=\text{Grad}\,U$. **(2001)**

Q9. Find the directional derivative of $f=x^2yz^3$ along $x=e^{-t}$, $y=1+2\sin t$, $z=t-\cos t$ at $t=0$. **(2001)**

Q10. Show that $\vec{\nabla}\times\frac{(\vec{a}\times\vec{b})}{r^3}=-\frac{(\vec{a}\times\vec{r})}{r^3}=-\frac{\vec{a}}{r^3}+\frac{3\vec{r}}{r^5}(\vec{a}\cdot\vec{r})$, where $\vec{a}$ is any constant vector.

(2001)

Q11. Show that Curl $\left(\text{curl }\vec{V}\right) = \text{Grad}\left(\text{div}\vec{V}\right) - \nabla^2\vec{V}$ **(2002)**

Q12. Prove that the divergence of a vector field is invariant with respect to coordinate transformations. **(2003)**

Q13. Prove the identity: $\vec{\nabla}\left(A^2\right) = 2\left(\vec{A}\cdot\vec{\nabla}\right)\vec{A} + 2\vec{A}\times\left(\vec{\nabla}\times\vec{A}\right)$, where $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.

(2003)

Q14. Prove the identity: $\vec{\nabla}\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} + \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} + \vec{B}\times\left(\vec{\nabla}\times\vec{A}\right) + \vec{A}\times\left(\vec{\nabla}\times\vec{B}\right)$ **(2004)**

Q15. Show that if $\vec{A}$ & $\vec{B}$ are irrotational, then $\vec{A}\times\vec{B}$ is solenoidal. **(2004)**

Q16. Prove that the curl of a vector field is independent of the choice of coordinates. **(2005)**

Q17. Show that $\vec{\nabla}\times\left(\hat{k}\times grad\frac{1}{r}\right) + grad\left(\hat{k}\cdot grad\frac{1}{r}\right) = 0$, where r is the distance from the origin and $\vec{k}$ is the unit vector in the direction $0Z$. **(2005)**

Q18. Find the values of constants a, b and c so that the directional derivative of the function $f = axy^2 + byz + cz^2x$ at the point $1, 2, -1$ has maximum magnitude 64 in the direction parallel to z - axis. **(2006)**

Q19. Prove that $r^n\vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if $n + 3 = 0$.

(2006)

Q20. If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}$, $r = |\vec{r}|$ determine $\vec{\nabla}\left(\frac{1}{r}\right)$ in terms of $\hat{r}$ and r. **(2007)**

Q21. For any constant vector $\vec{a}$, show that the vector represented by $\vec{\nabla}\times\left(\vec{a}\times\vec{r}\right)$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z) measured from the origin. **(2007)**

Q22. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find the value(s) of n in order that $r^n\vec{r}$ may be (i) Solenoidal (ii) Irrotational. **(2007, 2011)**

Q23. Prove that $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ where $r = \left(x^2 + y^2 + z^2\right)^{1/2}$. Hence find $f(r)$ such that $\nabla^2 f(r) = 0$. **(2008)**

Q24. Show that $\vec{\nabla} \cdot \left(\vec{\nabla} r^n\right) = n(n+1) r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$. **(2009)**

Q25. Find the directional derivative of

(i) $4xz^3 - 3x^2 y^2 z^2$ at $(2,-1,2)$ along Z - axis.

(ii) $x^2 yz + 4xz^2$ at $(1,-2,1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(2009)**

Q26. Find the directional derivative of $f(x,y) = x^2 y^3 + xy$ at the point $(2,1)$ in the direction of a unit vector which makes an angle of $\frac{\pi}{3}$ with the x - axis. **(2010)**

Q27. Prove that $\vec{\nabla} \cdot \left(f\vec{\nabla}\right) = f\left(\vec{\nabla} \cdot \vec{V}\right) + \left(\vec{\nabla} f\right) \cdot \vec{V}$, where f is a scalar function. **(2010)**

Q28. If u and v are two scalar fields and $\vec{f}$ is the vector field such that $u\vec{f} = \vec{\nabla}(V)$, find the value of $\vec{f} \cdot \left(\vec{\nabla} \times \vec{f}\right)$.

(2011)

Q29. Calculate $\nabla^2\left(r^n\right)$ and find its expression in terms of vector, r being the distance of any point (x,y,z) from the origin, n being constant and ∇^2 being Laplace operator. **(2013)**

CIVIL SERVICES EXAMINATION (MAINS)

Unit 29: Curvature and Torsion

Q1. Find the length of the arc of the twisted curve $\vec{r} = \left(3t, 3t^2, 2t^3\right)$ from the point $t = 0$ to the point $t = 1$. Find also the unit tangent t, unit normal n and the unit binormal b at $t = 1$. **(2001)**

Q2. Find the curvature k for the space curve:

$$x = a\cos\theta, \quad y = a\sin\theta, \quad z = a\theta\tan\alpha$$ **(2002)**

Q3. Find the radii of curvature and torsion at a point of intersection of the surfaces $x^2 - y^2 = c^2$, $y = x\tanh\frac{x}{c}$. **(2003)**

Q4. Show that the Frenet-Serret Formula can be written in the form

$$\frac{d\vec{T}}{dS} = \vec{\omega}\times\vec{T}, \quad \frac{d\vec{N}}{dS} = \vec{\omega}\times\vec{N}, \quad \frac{d\vec{B}}{dS} = \vec{\omega}\times\vec{B} \text{ where } \vec{\omega} = \tau\vec{T} - k\vec{B}$$ **(2004)**

Q5. Find the curvature and the torsion of the space curve

$$x = a\left(3u - u^3\right),\ y = 3au^2,\ z = a\left(3u + u^2\right).$$ **(2005)**

Q6. The parametric equation of a circular helix is $\vec{r} = a\cos u\hat{i} + a\sin u\hat{j} + cu\hat{k}$, where c is a constant and u is a parameter. Find the unit tangent vector $\hat{t}$ at the point u and the arc length measured from $u = 0$. Also find $\frac{d\hat{t}}{dS}$ where S is the arc length. **(2005)**

Q7. If the unit tangent vector $\vec{t}$ and binomial $\vec{b}$ makes angles θ & ϕ respectively with a constant unit vector $\vec{a}$, prove that $\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = -\frac{k}{\tau}$. **(2006)**

Q8. Find the curvature and torsion at any point of the curve:

$$x = a\cos 2t \qquad y = a\sin 2t \qquad z = 2a\sin t.$$ **(2007)**

Q9. Show that for the space curve

$$x = t,\ y = t^2\ z = \frac{2}{3}t^3$$

The curvature and torsion are same at every point. **(2008)**

Q10. Find $\frac{k}{\tau}$ for the curve $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + bt\,\hat{k}$ **(2010)**

Q11. Derive the Frenet-Serret Formulae. Define the curvature and torsion for a space curve. Compute them for the space curve

$$x = t, \quad y = t^2 \quad z = \frac{2}{3}t^3.$$

Show that the curvature and torsion are equal for the curve. **(2012)**

Q12. A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\,\hat{j} - t\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. **(2013)**

Q13. Show that the curve $\vec{x}(t) = t\hat{i} + \frac{1+t}{t}\hat{j} + \frac{1-t^2}{t}\hat{k}$ lies in a plane. **(2013)**

Q14. Find the curvature vector at any point of the curve $\vec{r}(t) = t\cos t\hat{i} + t\sin t\hat{j},\ 0 \le t \le 2\pi$. Give its magnitude. **(2014)**

CIVIL SERVICES EXAMINATION (MAINS)

Unit 30: Line, Surface and Volume Integrals

Q1. Evaluate $\iint_S \vec{\nabla}\times\vec{F}\cdot\hat{n}\,dS$ where S is the upper half surface of the unit sphere $x^2+y^2+z^2=1$ and $\vec{F}=z\hat{i}+x\hat{j}+y\hat{k}$ **(1993)**

Q2. If $\vec{F}=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$ evaluate $\iint \vec{\nabla}\times\vec{F}\cdot\hat{n}dS$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy- plane. **(1994)**

Q3. Let the region V be bounded by the smooth surface S and let n denote outward drawn unit normal vector at a point on S, if ϕ is harmonic in V, then show that $\int_S \frac{\partial\phi}{\partial n}\cdot dS$ is 0. **(1995)**

Q4. Verify Gauss's divergence theorem for $\vec{F}=xy\hat{i}+z^2\hat{j}+2yz\hat{k}$ on the tetrahedron $x=y=z=0,\ x+y+z=1$. **(1996)**

Q5. Verify Gauss's theorem for $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=4,\ z=0\ \&\ z=3$. **(1997)**

Q6. Evaluate by Green's theorem: $\int_C e^{-x}\sin y\,dx+e^{-x}\cos y\,dy$, where C is rectangle whose vertices are $(0,0)$, $(\pi,0)$, $\left(\pi,\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$. **(1999)**

Q7. Evaluate $\iint_S \vec{F}\cdot\vec{n}\,ds$ where $\vec{F}=2xy\hat{i}+yz^2\hat{j}+xz\hat{k}$ and S is the surface of the parallel piped bounded by $x=0,\ y=0,\ z=0$ and $x=2,\ y=1\ \&\ z=3$. **(2000)**

Q8. Verify Gauss's divergence theorem for $\vec{A}=(4x,-2y^2,z^2)$ taken over the region bounded by $x^2+y^2=4,\ z=0\ \&\ z=3$. **(2001)**

Q9. Let D be a closed and bounded region having boundary S. Further, let f be a scalar function having second order partial derivatives defined on it. Show that

$$\iint_S (f\,\text{grad}\,f)\cdot\hat{n}\,dS=\iiint_V\left[|\text{grad}\ f|^2+f\nabla^2 f\right]dV$$

hence or otherwise evaluate: $\iint_S (f \operatorname{grad} f)\cdot \hat{n}\, dS$ for $f = 2x + y + 2z$ over $x^2 + y^2 + z^2 = 4$. **(2002)**

Q10. Evaluate $\iint_S \operatorname{curl} \vec{A}\cdot d\vec{S}$, where S is the open surface $x^2 + y^2 - 4x + 4z = 0,\ z \ge 0$ and $\vec{A} = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$. **(2003)**

Q11. Derive the identity: $\iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, dV = \iint_S (\phi\nabla\psi - \psi\nabla\phi)\cdot \hat{n}\, dS$, where V is the volume bounded by the closed surface S. **(2004)**

Q12. Verify Stoke's theorem for $\vec{f} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **(2004)**

Q13. Evaluate $\iint_S x^3 dydz + x^2 y dzdx + x^2 z dxdy$ by Gauss's divergence theorem, where S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by $z = 0$ & $z = b$. **(2005)**

Q14. Verify Stoke's theorem for the function: $\vec{F} = x^2\hat{i} - xy\hat{j}$, integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0,\ y = 0,\ x = a$ & $y = a,\ a > 0$. **(2006)**

Q15. Determine $\int_C ydx + zdy + xdz$ by using Stoke's theorem where C is the curve defined by $(x - a)^2 + (y - a)^2 + z^2 = 2a^2,\ x + y = 2a$ that starts from the point $(2a, 0, 0)$ and goes at first below the z - plane. **(2007)**

Q16. Evaluate $\int_C \vec{A}\cdot d\vec{r}$ along the curve $x^2 + y^2 = 1,\ z = 1$ from $(0,1,1)$ to $(1,0,1)$ if $\vec{A} = (yz + 2x)\hat{i} + xz\hat{j} + (xy + 2z)\hat{k}$. **(2008)**

Q17. Evaluate $\iint_V \vec{F}\cdot\hat{n}dS$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface of the cylinder boundary by $x^2 + y^2 = 4$, $Z = 0$ and $Z = 3$. **(2008)**

Q18. Using divergence theorem, evaluate $\iint \vec{A}\cdot\hat{n}dS$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(2009)**

Q19. Find the value of $\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane $z=0$ when $\vec{F}=(y^2+z^2-x^2)\hat{i}+(z^2+x^2-y^2)\hat{j}+(x^2+y^2-z^2)\hat{k}$. **(2009)**

Q20. Use the divergence theorem to evaluate $\iint_S \vec{V}\cdot\hat{n}dA$ where $\vec{V}=x^2z\hat{i}+y\hat{j}-xz^2\hat{k}$ and S is the boundary of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4y$. **(2010)**

Q21. Verify the Green's theorem for $e^{-x}\sin y dx+e^{-x}\cos y dy$, the path of integration being the boundary of the square whose vertices are $(0,0)\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$. **(2010)**

Q22. If $\vec{u}=4y\hat{i}+x\hat{j}+2z\hat{k}$ calculate the $\iint_S (\vec{\nabla}\times\vec{u})\cdot d\vec{S}$ over the hemisphere given by $x^2+y^2+z^2=a^2,\quad z\geq 0$. **(2011)**

Q23. Verify the Gauss's divergence theorem for the vector $\vec{u}=x^2\hat{i}+y^2\hat{j}+z^2\hat{k}$ taken over the cube $x,y,\geq 0,\qquad z^2\leq 1$. **(2011)**

Q24. Verify Green's theorem in the plane for $\oint_C (xy-y^2)dx+x^2dy$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. **(2012)**

Q25. If $\vec{F}=y\hat{i}+(x-2xz)\hat{j}-xy\hat{k}$, evaluate $\iint_S (\vec{\nabla}\times\vec{F})\cdot\hat{n}d\vec{S}$, where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy plane. **(2012)**

Q26. By using divergence theorem of Gauss, evaluate the $\iint_S (a^2x^2+b^2y^2+c^2z^2)^{-1/2}\cdot dS$, where S is the surface of the ellipsoid $ax^2+by^2+cz^2=1$ and a,b and c being all positive constants. **(2013)**

Q27. Using Stoke's theorem to evaluate the $\int_C -y^3dx+x^2dy-z^3dz$, where C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. **(2013)**

Q28. Evaluate by Stoke's theorem: $\int_{\Gamma} ydx + zdy + xdz$, where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the z - plane. **(2014)**

CIVIL SERVICES EXAMINATION (MAINS)

Unit 31: Simple Harmonic Motion (Motion in a Plane)

Q1. A particle moving with uniform acceleration describes distances S_1 and S_2 metres in successive intervals of time t_1 and t_2 seconds. Express the acceleration in terms of S_1, S_2, t_1 and t_2. **(2004)**

Q2. A particle whose mass is m, is acted upon by a force $m\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at origin in time $\frac{\pi}{4}$. **(2006, 2012)**

Q3. A particle is performing simple harmonic motion of period T about a centre O. It passes through a point $p(op=p)$ with velocity v in the direction op. Show that the time which elapses before it returns to P is $\frac{T}{\pi}\tan^{-1}\frac{VT}{2\pi p}$. **(2007)**

Q4. One end of a light elastic string of natural length l and modulus of elasticity 2 mg is attached to a fixed point O and the other end to a particle of mass m. The particle initially held at rest at O is let fall. Find the greatest extension of the string during the motion and show that the particle will reach O again after a time $\left(\pi+2-\tan^{-1}2\right)\sqrt{\frac{2l}{g}}$.

(2009)

Q5. (i) After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half of its velocity. If it now reaches the ground in 1 second, find the height of glass above the ground. **(2011)**

(ii) A particle of mass m moves on straight line under an attractive force mn^2x towards a point O on the line, where x is the distance from O. If $x=a$ and $\frac{dx}{dt}=u$ when $t=0$, find $x(t)$ for any time $t>0$. **(2011)**

Q6. The velocity of a train increases from 0 to v at a constant acceleration f_1, then remains constant for an interval and again decreases to 0 at a constant retardation f_2. If the total distance described is x, find the total time taken. **(2011)**

Q7. A particle is performing a simple harmonic motion (SHM) of a period T about a centre O with amplitude a and it passes through a point P, where $OP = b$ in the direction of OP. Prove that the time which elapses before it returns to P is $\frac{T}{\pi}\cos^{-1}\frac{b}{a}$. **(2014)**

Q8. A particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μy^2 and when $y = a$, it is projected parallel to the axis of x with velocity $\sqrt{\frac{2m}{a}}$. Find the parametric equation of the path of the particle. Here μ is a constant. **(2014)**

CIVIL SERVICES EXAMINATION (MAINS)

Unit 32: Projectile Motion

Q1. Prove that the velocity required to project a particle from a height h to fall at a horizontal distance a from a point of projection is at least equal to $\sqrt{g\sqrt{a^2+h^2-h}}$. **(2004)**

Q2. If V_1, V_2, V_3 are the velocities at three points A, B, C of the path of a projectile, where the inclinations to the horizon are α, $\alpha-\beta$, $\alpha-2\beta$ and if t_1, t_2 are the times of describing the arcs AB, BC respectively, prove that $V_3t_1 = V_1t_2$ and $\frac{1}{V_1}+\frac{1}{V_3}=\frac{2\cos\beta}{V_2}$. **(2010)**

Q3. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a meter short of it when the angle of projection is α and goes y meter beyond when the angle of projection is β. If the velocity of projection is assumed same in all cases, find the correct angle of projection. **(2011)**

CIVIL SERVICES EXAMINATION (MAINS)

Unit 33: Constrained Motion

Q1. If a particle slides down a smooth cycloid, starting from a point whose actual distance from the vertex is b, prove that its speed at any time t is $\frac{2xb}{T}\sin\left(\frac{2xT}{T}\right)$, where T is the time of complete oscillation of the particle. **(2003)**

Q2. A particle is projected along the inner side of a smooth vertical circle of radius a so that velocity at the lowest point is u. Show that $2ag < u^2 < 5ag$. The particle will the highest point and will describe a parabola whose latus rectum is $\frac{2\left(u^2 - 2ag\right)^3}{27a^2g^3}$.

(2005)

Q3. Two particles connected by a fine string are constrained to move in a fine cylendrial tube in a vertical plane. The axis of the cycloid is vertical with vertex upwards. Prove that the tension in the string is constant throughout motion. **(2005)**

Q4. A particle is free to move on a smooth vertical circular wire of radius a. It is projected horizontally from the lowest point with velocity $2\sqrt{ag}$. Show that the reaction between the particle and the wire is zero after a time $\sqrt{\frac{a}{g}}\log\left(\sqrt{5}+\sqrt{6}\right)$. **(2006)**

Q5. A particle is projected with velocity v from the cusp of a smooth inverted cycloid down the arc. Show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\cot^{-1}\frac{v}{2\sqrt{ag}}$. **(2009)**

Unit 34: Central Orbits

Q1. A particle of mass m moves under a force $m\mu\left\{3au^4 - 2\left(a^2 - b^2\right)u^5\right\}$, $u = \frac{1}{r}$, $a > b$ and $\mu > 0$ being given constants. It is projected from as apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is given by $r = a + b\cos\theta$, where $\left(r, \theta\right)$ are the plane polar coordinates of a point. **(2008)**

Q2. A body is describing an ellipse of eccentricity e under the action of a central force directed towards a focus and when at the nearer apse, the centre of force is transferred to other focus. Find the eccentricity of the new orbit in terms of the original orbit. **(2009)**

Q3. A particle moves with a central acceleration $\mu\left(r^5 - 9r\right)$ being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{2u}$. Show that its path is $x^4 + y^4 = 9$. **(2010)**

Unit 35: Work, Energy and Impulse

Q1. A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $=\sqrt{\frac{2mE}{M(M+m)}}$ and the initial velocity of the shot is $\sqrt{\frac{2ME}{M(M+m)}}$.

Q2. A shell of mass M is moving with velocity v. An internal explosion generates an amount of energy E and breaks the shell into two portions whose masses are in the ratio $m_1 : m_2$. The fragments continue to move in the original line of motion of the shell. Show that their velocities are $v+\sqrt{\frac{2m_2E}{m_1M}}$ and $v-\sqrt{\frac{2m_1E}{m_2M}}$.

Q3. A bullet of mass m moving with velocity v, strikes a block of mass M, which is free to move in the direction of the motion of the bullet and is embedded in it. Show that a portion $\frac{M}{M+m}$ of the K.E. is lost. If the block is afterwards struck by an equal bullet moving in the same direction with the same velocity. Show that there is a further loss of K.E. equal to $\frac{mM^2v^2}{2(m+M)(M+2m)}$.

Q4. A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Prove that the velocity of the recoil is $\left[\frac{2m^2gh}{M(M+m)}\right]^{1/2}$.

Q5. A train of mass M lb is ascending a smooth incline of 1 in n and when the velocity of the train is v ft/sec, its acceleration is f ft/sec^2. Prove that the effective HP of the engine is $\frac{Mv(nf+g)}{550ng}$.