

CIVIL SERVICES EXAMINATION (MAINS) 2022**PHYSICS PAPER - I: MECHANICS****TUTORIAL SHEET: 1****Conservation laws**

1. Define a conservative field. Determine if the field given below is conservative in nature: where c is a constant. $E = c [y^2 \hat{i} + (2xy + z^2) \hat{j} + 2yz \hat{k}]$ volts per meter, where c is a constant. (2012)
2. If the forces acting on a particle are conservative, show that the total energy of the particle which is the sum of the kinetic and potential energies is conserved. (2013)
3. Prove that as a result of an elastic collision of two particles under non – relativistic regime with equal masses, the scattering angle will be 90° . Illustrate your answer with a vector diagram. (2013)
4. Discuss the problem of scattering of charged particle by a coulomb field. Hence, obtain an expression for Rutherford scattering cross-section. What is the importance of the above expression? (2014)
5. Write down precisely the conservation theorems for energy, linear momentum and angular momentum of a particle with their mathematical forms. (2015)
6. Draw a neat diagram to explain the scattering of n incident beam of particles by a centre of force. (2015)
7. Show that the differential scattering cross-section can be expressed as $\sigma(\theta) = \frac{s}{\sin \theta} \left| \frac{ds}{d\theta} \right|$ where s is the impact parameter and θ is the scattering angle. (2015)
8. (i) The distance between the centres of the carbon and oxygen atoms in the carbon monoxide (CO) gas molecule is 1.130×10^{-10} m. Locate the centre of mass of the molecule relative to the carbon atom.
(ii) Find the centre of mass of a homogeneous semicircular plate of radius a . (2016)
9. A diatomic molecule can be considered to be made up of two masses m_1 and m_2 separated by a fixed distance r . Derive a formula for the distance of centre of mass, C , from mass m_1 . Also show that the moment of inertia about an axis through C and perpendicular to r is μr^2 , where $\mu = \frac{m_1 m_2}{m_1 + m_2}$. (2017)
10. A ball moving with a speed of 9 m/s strikes an identical stationary ball such that after the collision the direction of each ball makes an angle 30° with the original line of motion. Find the speed of the balls after the collision. Is the kinetic energy conserved in this collision? (2017)
11. (a) If a particle of mass m is in a central force field $f(r) \hat{r}$, then show that its path must be a plane curve where \hat{r} is a unit vector in the direction of position vector \vec{r} .
(b) A block of mass m having negligible dimension is sliding freely in x -direction with velocity: $\vec{v} = v \hat{i}$ as shown in the diagram. What is its angular momentum \vec{L}_O about origin O and its angular momentum \vec{L}_A about the point A on y -axis? (2018)
12. (i) What is central force? Give two examples of the central force.

(ii) Show that the angular momentum (\vec{L}) of the particle in a central force field is a constant of motion. **(2019)**

13. A proton is 1837 times heavier than an electron. Find the centre of mass of hydrogen atom. **(2019)**

14. Show that the cross-section for elastic scattering of a point particle from an infinitely massive sphere of radius R is $\frac{R^2}{4}$. What is the inference of this result? **(2019)**

15. A rocket starts vertically upwards with speed v_0 . Then define its speed v at a height h in terms of v_0 , h , R (radius of Earth) and g (acceleration due to gravity on Earth's surface). Also calculate the maximum height attained by a rocket fired with a speed of 90% of the escape velocity. **(2020)**

TUTORIAL SHEET: 2
Central Force Motion and Gravitation

1. A planet revolves around the sun in an elliptic orbit of eccentricity e . If T is the time period of the planet, find the time is spent by the planet between the ends of the minor axis close to the sun. **(2010)**
2. A particle is moving in a central force field $r = ke^{\alpha\theta}$
(i) Find the force law (ii) Find $\theta(t)$ (iii) Find the total energy **(2012)**
3. A particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of distance. **(2013)**
4. A charge particle is moving under the influence of a point nucleus. Show that the orbit of the particle is an ellipse. Find out the time period of the motion. **(2014)**
5. The density inside a solid sphere of radius a is given by $\rho = \frac{\rho_0 a}{r}$, where ρ_0 is the density at the surface and r denotes the distance from the centre. Find the gravitational field due to this sphere at a distance $2a$ from its centre. **(2014)**
6. Draw a neat diagram to explain the scattering of an incident beam of particles by a center of force. **(2015)**
7. A body moving in an inverse square attractive field traverses on elliptical orbit with eccentricity e and period γ . Find the time taken by the body to traverse the half of the orbit that is nearer the center of force. Explain briefly why a comet spends only 18% of its time on the half its orbit that is nearer the sun. **(2016)**
8. Express angular momentum in terms of kinetic, potential and total energy of a satellite of mass m in a circular orbit of radius r . **(2017)**

TUTORIAL SHEET: 3

Rotating Frames of Reference

1. A rigid body is spinning with an angular velocity ω of 4 radian/sec about an axis parallel to the direction $(4\mathbf{j} - 3\mathbf{k})$ passing through the point A with $\mathbf{OA} = (2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ where O is the origin of the coordinate system. Find the magnitude and direction of the linear velocity of the body at point P with $\mathbf{OP} = (4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. (2012)
2. Suppose that as S' - frame is rotating with respect with respect to a fixed frame having the same origin. Assume that the angular velocity ω of the S' - frame is given by $\omega = 2t\mathbf{i} - t^2\mathbf{j} + (2t + 4)\mathbf{k}$. Where t is time and the position vector \mathbf{r} of a typical particle at time t as assumed in S' - frame is given by $\mathbf{r} = (t^2 + 1)\mathbf{i} - 6t\mathbf{j} + 4t^3\mathbf{k}$. Calculate the Coriolis acceleration at $t=1$ second. (2013)
3. Calculate the horizontal component of the Coriolis force acting on a body of mass 0.1 kg moving north ward with a horizontal velocity of 100 ms^{-1} at 30° N latitude on the earth. (2013)
4. Derive the expression for Coriolis force and show that this force is perpendicular to the velocity and to the axis of rotation. What is the nature of this force? (2016)

TUTORIAL SHEET: 4

RIGID BODY DYNAMICS

1. A uniform solid sphere of radius R having moment of inertia I about its diameter is melted to form a uniform moment of inertia of the disc about an axis passing through its edge and perpendicular to the plane is also equal to I . Show that the radius r of the disc is given by $r = \frac{2R}{\sqrt{15}}$. (2010)
2. Show that the kinetic energy and angular momentum of torque free motion of a rigid body is constant. (2013)
3. If I^1 and I be the moments of inertia of a body about an axis passing through an arbitrary origin and about a parallel axis through the centre of mass respectively, show that $I^1 = MR^2 + I$, where R is the position vector of the centre of mass with respect to the arbitrary origin and M is the mass of the body. (2014)
4. Consider a rigid body rotating about an axis passing through a fixed point in the body with an angular velocity ω . Determine the kinetic energy of such a rotating body in a coordinate system of principal axis. If the earth suddenly stops rotating, what will happen to the rotational kinetic energy? Comment in detail. (2014)
5. A body turns a fixed point. Show that the angle between its angular velocity vector and its angular momentum vector about a fixed point is always acute. (2014)
6. How does one obtain the angular velocity of the Earth about the North Pole with respect to a fixed star as $7.292 \times 10^{-5} \text{ sec}^{-1}$? Explain your method of calculating the above value. (2015)
7. Show that the moment of inertia of a circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}MR^2$ (2015)
8. Four solid spheres A,B,C, and D each of mass m and radius a , are placed with their centres on the four corners of square of side b as shown in the figure below:
9. Calculate the moment of inertia of the system about one side of the square, Also calculate the moment of inertia of the system about a diagonal of the square. (2016)
10. Define moment of inertia and explain its physical significance .Calculate the moment of inertia of an annular ring about an axis passing through its centre and perpendicular to its plane. (2017)
11. Find the moments of inertia of rigid diatomic molecule about different axes of symmetry through the centre of mass. (2019)

12. Write down Euler's dynamical equations of motion (no derivation) of a rigid body about a fixed point under the action of a torque. Show that the kinetic energy of the torque-free motion is constant.

(2019)

13. Where do you find the applications of gyroscope?

A top of mass 0.200 kg is made up of a thin disc of radius 0.12 m. It is pierced in the centre and a pin of negligible mass is mounted normal to its plane. The pivot under the disc is 0.03 m long. The tip is made to spin with its axis making an angle $\theta = 20^\circ$ with the vertical and a precessional angular speed of 2 rad/s. Calculate the angular speed with which it spins.

(2019)

14. Determine the location of the centre of mass of a uniform solid hemisphere of radius R and mass M from the centre of its base.

(2020)

15. Obtain expressions for the moment of inertia of a solid cone about its

(i) Vertical axis and (ii) axis passing through the vertex and parallel to its base.

(2020)

TUTORIAL SHEET: 5

Mechanics of Continuous media

1. When a sphere of radius r falls down a homogeneous viscous fluid of unlimited extent with the terminal velocity v , the retarding viscous force acting on the sphere depends on the coefficient of viscosity η , the r and its velocity v . Show how Stokes law was arrived at connecting these quantities from the dimensional considerations. (2010)

2. Using Poiseuille's formula, show that the volume of a liquid of viscosity coefficient η passing per second through a series of two capillary tubes of lengths l_1 and l_2 having radii r_1 and r_2 is obtained as

$$Q = \frac{\pi p}{8\eta} \left[\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4} \right] \text{ where } p \text{ is the effective pressure difference across the series.} \quad (2015)$$

3. Define coefficients of viscosity and kinematic viscosity of a fluid. What are Poise and Stokes? (2015)
4. Write down Poiseuille's formula and mention its limitations in analyzing the flow of a liquid through a capillary tube. (2015)
5. Show that the Young's modulus Y , modulus of rigidity η and Poisson's ratio σ are related by the equation $Y=2\eta(1+\sigma)$. (2016)
6. A horizontal pipe of non-uniform bore has water flowing through it such that the velocity of flow is 40 cm/s at a point where the pressure is 2 cm of mercury column. What is the pressure at a point where the velocity of flow is 60 cm/s? (density of water = 1 g /c.c.) (2016)
7. State and explain Stokes' law. A drop of water of radius 0.01 m is falling through a medium whose density is 1.21 kg/m³ and $\eta = 1.8 \times 10^{-5}$ N-s/m². Find the terminal velocity of the drop of water. (2017)
8. In a horizontal pipeline of uniform area of cross-section, the pressure falls by 5 N m⁻² between two
9. points separated by a distance of 1 km. Calculate the change in kinetic energy per kg of oil flowing at these points. Density of oil = 800 kg m⁻³. (2019)
13. A rubber cord 1mm in diameter and 1 m long is fixed at one end and a weight of 1 kg is attached to the other end. If the Young's modulus of rubber is 0.05×10^{11} cm⁻², then find the period of the vertical oscillations of the weight. (2020)
14. A shaft of diameter 8 cm and length 5 m is transmitting power of 8 k W at 300 revolutions per minute. If the coefficient of rigidity of the material of the shaft be 8×10^{11} dynes/cm², then calculate the relative shift between the ends of the shaft. (2020)

TUTORIAL SHEET: 6

Special Theory of Relativity

1. What is the significance of the null result of Michelson-Morley experiment? Does it disprove the existence of ether? Justify. (2010)
2. A particle of rest mass $M = 4 \times 10^{-27} \text{ kg}$, disintegrates into two particles of rest masses $M_1 = 3 \times 10^{-27} \text{ kg}$ and $M_2 = 1 \times 10^{-27} \text{ kg}$. Show that the energies E_1 and E_2 of these two parts after disintegration satisfy the condition $E_1 = 3 E_2$ while moving in opposite direction with equal linear momenta, Give necessary mathematical derivation. (2013)
3. Show that the operator $\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right)$ is invariant under Lorentz transformations. (2013)
4. Show that a particle of rest mass m_0 , total energy E and linear momentum \mathbf{p} satisfies the relation
$$E^2 = c^2 \mathbf{p}^2 + m_0^2 c^4, \quad \text{Where } c \text{ is the velocity of light in free space.}$$
(2013)
5. Derive the relativistic length contraction using Lorentz transformation. (2013)
6. Prove mathematically that the addition of any velocity of a particle to the velocity of light in free space merely reproduces the velocity of light in free space only. (2015)
7. Show that the rest mass energy of an electron is 0.51 MeV .(use the standard values of the physical parameters). (2015)
8. Calculate the percentage contraction in the length of a rod in a frame of reference, moving with velocity $0.8c$ in a direction (i) parallel to its length and (ii) at an angle of 30° with its length. What is the orientation of the rod in the moving frame of reference in case (ii)? (2016)
9. Given proton for which $\beta = 0.995$ measured in the laboratory. What are the corresponding relativistic energy and momentum? Take, $m_p = 1.67 \times 10^{-24} \text{ g}$. (2016)
10. Describe Michelson-Morley experiment and show how the negative results obtained from this experiment were interpreted. (2017)
11. Prove that $x^2 + y^2 + z^2 = c^2 t^2$ is invariant under Lorentz transformation. (2017)
12. A reference frame S' moves with respect to rest frame S with a uniform velocity ' v ' parallel to x -direction. Show from Lorentz transformation that two events simultaneous ($t_1 = t_2$) at different positions ($x_1 \neq x_2$) in S frame are not in general simultaneous in S' frame. (2019)
13. The mean life of π meson is $2 \times 10^{-8} \text{ s}$. Calculate the mean life of a meson moving with a velocity of $0.8 c$, where c is the velocity of light. (2019)
14. Two β -particleless A and B emitted by a radioactive source R travel in opposite directions, each with a velocity of $0.9 c$ with respect to the source. Find the velocity of B with respect to A (Here c is the velocity of light). (2019)

30. What do you understand by length contraction ? Calculate the percentage length contraction of a rod moving with a velocity $0.8\ c$ in a direction at 60° with respect to its own length. (2020)
31. Derive the relativistic expression for kinetic energy by considering mass variation with velocity. Hence, establish the relation between momentum (p) and energy (E) for a relativistic particle; $\frac{dE}{dp} = v$. (2020)

WAVES & OPTICS/ TUTORIAL SHEET: 7A**SHM : Damped and Forced Oscillations**

1. Why do we prefer to work with a critically damped ballistic galvanometer in a laboratory? What is external critical damping resistance? (2019)

2. The equation for displacement (X) of a point on a damped oscillator is given by

$$X = 5 e^{-0.25t} \sin \left(\frac{\pi}{2} \right) t \text{ meters.}$$

Find velocity of oscillating at $t = \frac{T}{4}$ and T, where T is the time period of the oscillator. What is the direction of velocity in each case? (2020)

TUTORIAL SHEET: 7B**Beats, Stationary waves, Phase & Group velocity, Huygen 's Principle**

1. In the propagation of longitudinal waves in a fluid contained in an infinitely long tube of cross-section A, show that $\rho = \rho_0 \left(1 - \frac{\partial \xi}{\partial x} \right)$ Where , ρ_0 = equilibrium density ρ = density of the fluid in the disturbed state $\frac{\partial \xi}{\partial x}$ = volume strain $\left(\left| \frac{\partial \xi}{\partial x} \right| \ll 1 \right)$ (2010)
2. The dispersion relation for deep water waves is given by $\omega^2 = gk + ak^3$ where g and a are constants. Obtain expressions for phase velocity and group velocity in terms of the wavelength λ . ω and k represent the angular frequency and wave number respectively. (2013)
3. Show that the group velocity is equal to particle velocity. Also prove that the group velocity of the photons is equal to c, the velocity of light. (2014)
4. Find out the phase and group velocities of a radio wave of frequency $\omega = \sqrt{2}\omega_p$ in the ionosphere (as a dielectric medium) of refractive index $n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ Here, ω_p is the ionospheric plasma frequency. (2015)
5. The equation of a progressive wave moving on a string is $y = 5 \sin \pi (0.01 x - 2t)$. In this equation y and x are in centimeters and t is in seconds. Calculate amplitude, frequency and velocity of the wave. If two particles at any instant are situated 200 cm apart, what will be the phase difference between these particles? (2016)
6. Explain with proper example the interferences due to 'division of wavefront ' and 'division of amplitude'. (2017)
7. Find the velocity of sound in a gas in which two waves of wavelengths 1.00m and 1.01m produce 10 beats in 3 seconds. (2017)
8. When two waves of nearly equal frequencies interfere, then show that the number of beats produced per second is equal to the difference of their frequencies. (2018)

TUTORIAL SHEET: 8**Geometrical Optics**

1. Show that two convex lenses of the same material kept separated by a distance α which is equal to the average of two focal lengths, may be used as an achromat, that is, $\alpha = \frac{1}{2}(f_1 + f_2)$. (2010)
2. Using matrix method, find out the equivalent focal length for a combination of two thin lenses of focal lengths f and f separated by a distance a . (2015)
3. Obtain the system matrix for a thin lens placed in air and made of material of refractive index 1.5 having radius of curvature 50 cm each. Also find its focal length. (2017)
4. What do you mean by spherical aberration of a lens? Show that if two plano-convex lenses are kept at a distance equal to the difference of their focal lengths, the spherical aberration would be minimum. (2018)
5. What is axial chromatic aberration?
A convex lens has a focal length of 15.5×10^{-2} m for red colour and 14.45×10^{-2} m for violet colour. If an object is kept at a distance of 40 cm from the lens, calculate the longitudinal chromatic aberration of the lens. (2019)
6. Prove that when light goes from one point to another via a plane mirror, the path followed by light is the one for which the time of flight is the least. (2019)
14. State and explain Fermat's principle of extremum path. Discuss the cases of rectilinear propagation of light and reversibility of light rays in context of Fermat's principle. Using Fermat's principle, deduce the thin lens formula. (2020)

TUTORIAL SHEET: 9**Interference**

1. Describe Michelson interferometer for evaluation of coherence length of an optical beam. Calculate coherence length of a light beam of wavelength 600 nm with spectral width of 0.01 nm. (2010)
2. Show that two light beams polarized in perpendicular directions will not interfere. (2010)
3. An optical beam of spectral width 7.5 GHz at wavelength $\lambda=600$ nm is incident normally on Fabry-Perot etalon of thickness 100 mm. Taking refractive index unity find the number of axial modes which can be supported by the etalon. (2010)
4. In a Young double slit experiment, the first bright maximum is displaced by $y = 2$ cm from the central maximum. If the spacing between slits and distance from the screen are 0.1 mm and 1 m respectively, find the wavelength of light. (2013)
5. Obtain the conditions for constructive interference and destructive interference in a thin film due to reflected light. (2016)
6. In Michelson interferometer, 100 fringes cross the field of view when the movable mirror is displaced through 0.029 mm. Calculate the wavelength of the light source used. (2016)
7. What is multiple-beam interference? Discuss the advantages of multiple-beam interferometry over two-beam interferometry. Explain the fringes formed by Fabry - Perot interferometer. (2017)
8. Explain with proper example the interferences due to 'division of wavefront' and division of amplitude'. (2017)
9. Describe how Michelson Interferometer can be used to determine refractive index of a gas.
In a Michelson Interference experiment, a tube of length 25 cm containing a gas of refractive index μ is introduced between the upper mirror and the beam splitter. 150 fringes cross the centre of the field of view when the wavelength of light used is 5890 Å. Find the value of μ . (2018)
10. (i) What are the fringes of equal thickness and fringes of equal inclination?

(ii) In a Newton's ring arrangement with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-7}m$ and $\lambda_2 = 5.9 \times 10^{-7}m$, it is found that the m^{th} dark ring due to one wavelength coincides with the $(m + 1)^{th}$ dark ring due to the other. Find the diameter of the m^{th} dark ring, if the radius of curvature of the lens is 90 cm. (2019)
29. What are Newton's rings ? How are they formed by two curved surfaces ? (2020)
30. Discuss the conditions for interference. Describe Young's double-slit experiment and derive an expression for the estimation of fringe width. Discuss its dependency on various parameters.
Green Light of wavelength 5100 Å from a narrow slit is incident on a double-slit. If the overall separation of 10 fringes on a screen 200 cm away is 2 cm, find the slit separation. (2020)

TUTORIAL SHEET: 10**Diffraction**

1. Obtain the expression for the primary focal length of Fresnel zone plate. (2010)
2. A parallel beam of light from a He – Ne laser ($\lambda = 630 \text{ nm}$) is made to fall on a narrow slit of width $0.2 \times 10^{-3} \text{ m}$. The Fraunhofer diffraction pattern is observed on a screen placed in the focal plane of a convex lens of focal length 0.3 m . Calculate the distance between the (i) First two minima and (ii) first two maxima on the screen. (2013)
3. A convex lens of focal length 20 cm is placed after a slit of width 0.5 mm . If a plane wave of wavelength 5000 \AA falls normally on the slit, calculate the separation between the second minima on either side of the central maximum. (2015)
4. Show that the areas of all the half-period zones are nearly the same. Find the radius of 1st half-period zone in a zone plate whose focal length is 50 cm and the wavelength of the incident light is 500 nm . (2017)
5. Discuss the intensity distribution in Fraunhofer diffraction pattern due to a single slit. Obtain conditions for maxima and minima of the intensity distribution. Show that the intensity of the first maxima is about 4.95% of that of the principal maxima. (2018)
6. What is a zone plate ? Give its theoretical description . Show that a zone plate has multiple foci. Differentiate a zone plate from a convex lens. Calculate the radius of the first half period zone in a zone plate behaving like a convex lens of focal length 60 cm for light of wavelength 6000 \AA . (2020)

TUTORIAL SHEET :11**Diffraction Grating/Circular Aperture**

1. Explain the physical significance of resolving power of a grating with relevant mathematical expression. (2013)
2. Considering a plane transmission diffraction grating, where d is the distance between two consecutive ruled lines, m as the order number and θ as the angle of diffraction for normal incidence, calculate the angular dispersion $\frac{d\theta}{d\lambda}$ for an incident light of wavelength λ . (2014)
3. Can D_1 and D_2 lines of sodium light ($\lambda_{d1} = 5890 \text{ \AA}$ and $\lambda_{d2} = 5896 \text{ \AA}$) be resolved in second-order spectrum if the number of lines in the given grating is 450? Explain. (2016)
4. Obtain an expression for the resolving power of grating explaining the Rayleigh's criterion of resolution. (2016)
5. A plane transmission grating has 3000 lines in all, having width of 3 mm. What would be the angular separation in the first order spectrum of the two sodium lines of wavelengths 5890 Å and 5896 Å? Can they be seen distinctly? (2018)

TUTORIAL SHEET: 12**Polarization**

1. Find out the angle between the reflected and refracted rays when a parallel beam of light is incident on a dielectric surface at an angle equal to the Brewster's angle. Explain how do you use this concept to produce linearly polarized light. (2014)
2. Explain the principle of producing polarized light by the method of reflection, refraction and double refraction with the help of neat diagrams. (2016)
3. Sunlight is reflected from a calm lake. The reflected light is 100% polarized at a certain instant. What is the angle between the sun and horizon? (2017)
4. A plane-polarized light passes through a double-refracting crystal of thickness $40 \mu\text{m}$ and emerges out as circularly polarized. If the birefringence of the crystal is 0.00004, then find the wavelength of the incident light. (2017)
5. Distinguish between positive and negative crystals in terms of double refraction. How are these crystals used to make quarter wave plates? Explain how the quarter wave plate is used in producing elliptically and circularly polarized light. (2018)
6. (i) How can one convert a left-handed circularly polarized light into a right-handed one (and vice versa)?
(ii) Calculate the thickness of a quarter-wave plate when the wavelength of light is 589 nm.
Given: $\mu_o = 1.544$ and $\mu_E = 1.553$. (2019)
15. Explain the phenomenon of double refraction calcite crystal. Considering birefringent crystal as non-conducting material, explain double refraction using electromagnetic theory.
Calculate the thickness of a double refracting plate which produces a path difference of $\frac{\lambda}{4}$ between extraordinary and ordinary waves.
Given:
 $\lambda = 5890 \text{ \AA}$, $\mu_o = 1.53$, $\mu_e = 1.54$ (2020)

TUTORIAL SHEET: 13**LASERS**

1. A laser beam of 1 micrometer wavelength with 3 megawatts power of beam diameter 10 mm is focused by a lens of focal length 50 mm. Evaluate the electric field associated with the light beam at the focal point. (Dielectric permittivity of freespace, $\epsilon_0 = 8.8542 \times 10^{-12} C^2 / N - m^2$)

(2010)

2. What is the physical significance of Einstein's A-coefficient? Explain why it is more difficult to achieve Lasing action at X-ray wavelength than at infra-red wavelength. (2014)

3. Explain the working principle of a 3-level laser with a specific example. Comment on why the third level is needed. (2014)

4. What is the role of an optical resonator in a laser? Why does one prefer curved mirrors instead of plane mirrors in designing an optical resonator? (2015)

5. Using the concept of Einstein's A and B coefficients for a two-level atomic system under thermal equilibrium, determine the ratio of the number of atoms per unit volume in the two levels experiencing spontaneous and stimulated emission. How does the principle of population inversion lead to the gain mechanism in the active medium of the laser? (2015)

6. Explain the principle of (i) induced absorption (ii) spontaneous emission and (iii) stimulated emission.

Show that the ratio of Einstein's coefficients is given by (2016)

$$\frac{A}{B} = \frac{8\pi h\nu^3}{c^3}$$

7. How is laser light Different from ordinary light? Discuss the working principle of ruby laser. What role do chromium ions play in this process? (2017)

8. Explain the principle and working of He-Ne laser. What is the role of He gas? Why is it necessary to use narrow tube? How many longitudinal modes can be excited for an He-Ne laser in a cavity of length 30 cm and having half width of gain profile of laser material $2 \times 10^{-3} nm$? The emission wavelength is 6328 Å. (2018)

9. Discuss how population inversion is achieved in Ruby laser.

What is 'laser spiking'? Why does it occur? (2019)

10. Briefly discuss the postulates of Einstein to explain stimulated emission. Derive an expression for Einstein's A and B coefficients and show that the ratio of spontaneous versus stimulated emission is proportional to the third power of frequency of radiation. Why is it difficult to achieve laser action in higher frequency ranges such as X-rays?
Can there be a temperature at which the rates of spontaneous and stimulates emission are equal ? Illustrate with wavelength $\lambda = 5000 \text{ \AA}$. **(2020)**

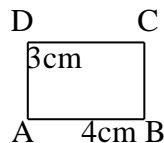
TUTORIAL SHEET: 14

Optical Fibres & Holography

1. What should be the refractive index of cladding of an optical fibre with numerical aperture 0.5 with refractive index of core as 1.5? **(2010)**
2. Explain why information carrying capacity of an optical fibre can be enhanced by reducing the pulse dispersion. How does one minimize pulse dispersion using a graded index optical fibre? **(2013)**
3. For a multimode step index optical fibre, the core refractive index is 1.5 and fractional index difference is 0.001. Calculate the pulse broadening for 1 km length of the fibre. Over a length of 2 km of the fibre, calculate the minimum pulse separation that can be transmitted without overlap. **(2014)**
4. How does holography differ from conventional photography? What are the requirements for the formation and reading of a hologram? **(2014)**
5. The refractive indices of core and cladding in a step index optical fiber are 1.52 and 1.48 respectively. The diameter of the core is 30 μm . If the operating wavelength is 1.3 μm , calculate the V parameter and the maximum number of modes supported by the fiber. **(2016)**
6. Explain the principal of operation of optical fibre. What are the different losses that take place in optical fibre? **(2017)**
7. In what way is holography different from conventional photography? Discuss the salient features of a hologram. What are the requirements for the formation and reading of a hologram? **(2019)**

ELECTRICITY & MAGNETISM**TUTORIAL SHEET: 15****Electrostatics**

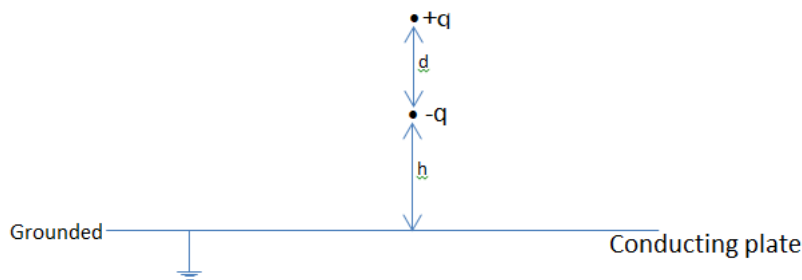
1. Obtain Poisson's equation in electro-statics from Gauss' law. What from does it takes the charge density is zero? (2010)
2. What is meant by a dielectric? Define polarization vector P and relate it with the average molecular dipole moment. Obtain expression for the potential due to a polarized dielectric in terms of the polarization vector. (2010)
3. Using Fundamentals concepts of electromagnetics in determine the electric field of an electron dipole \vec{P} at a distance \vec{r} and its energy in field \vec{E} (2013)
4. ABCD is a rectangle in which charges $+10^{-11}C$, $-2 \times 10^{-11}C$, and $10^{-11}C$ are placed at B, C, D respectively



Calculate the potential at the corner A and work done in carrying a charge of 2 coulomb to A (2013)

5. Under one-dimensional configuration, the charge density is given by $\rho(x) = \frac{\rho_0 x}{5}$; where ρ_0 is a constant charge density. If the electric field $|\vec{E}| = 0$ at $x = 0$ and potential $V = 0$ at $x = 5$, determine V and $|\vec{E}|$ (2015)
6. A conducting sphere of radius 5 cm has a total charge of $12nC$ uniformly distributed on its surface in free space. Determine the displacement vector \vec{D} on its surface and outside at a distance r from the center of the sphere. (2015)
7. With the help of a neat diagram, Show that the potential due to a dipole at a point is given by $V = \frac{1}{4\pi\epsilon_0} \frac{p \cos\theta}{r^2}$, where p is the dipole moment of the charge distribution, θ is the angle between the line joining the center of the dipole to the point of interest and the axis of the dipole. (2016)

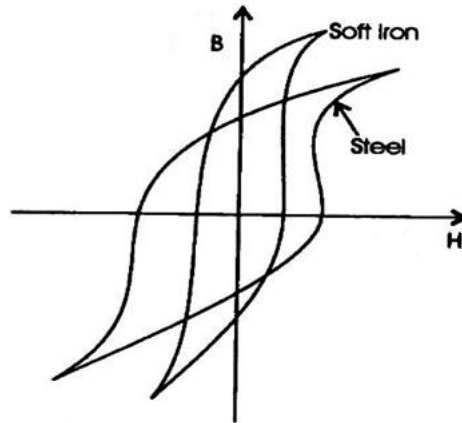
8. Find the capacitance of two concentric spherical metal shells having radii a and b . **(2019)**
9. Two conducting planes, intersecting at right-angles to each other, are kept at a potential ϕ_0 . Calculate the potential at a point in space if the total charge on a plane of area α be Q . **(2019)**
10. A vertically oriented electric dipole having dipole moment \vec{p} is kept at height h above an infinitely large horizontal conducting plate, which is grounded as shown in the diagram. Calculate the force between the electric dipole and the conducting plate by using method of images. **(2020)**



11. Write expressions for divergence and curl of an electrostatic field. From these, obtain Poisson and Laplace equations. Two concentric conducting spherical shells having radii r_1 and r_2 ($r_1 < r_2$) are charged to potentials V_1 and V_2 , respectively. What are the electric potential and hence electric field in the space between the shells? Also find the charge on the inner shell. **(2020)**

TUTORIAL SHEET: 16**Magnetostatics & Biot-Savart's law**

1. Based on the hysteresis loops for soft iron and steel as shown in the diagram, which material would you prefer to utilize for making transformer cores and why ? (2020)



TUTORIAL SHEET: 17 EMI & Alternating Current Circuits

1. Discuss the growth of current when an e.m.f. is suddenly applied to a circuit containing resistance, inductance and capacitance in series. What is the time constant of the circuit? (2010)
2. A series circuit has an inductance of 200 micro henries, a capacitance of 0.0005 microfarad and a resistance of 10 ohms. Find the resonant frequency and quality factor of the circuit. (2010)
3. A series LCR circuit has resonant frequency ω_0 and a large quality factor Q. Write down terms of R, ω , ω_0 and Q, its (i) impedance at resonance, (ii) impedance at half –power points and (iii) the approximate forms of its impedance at low and high frequencies. (2013)

4. Consider the following coupled inductor – capacitor circuit:

Calculate the ratio of the frequencies of the anti –symmetric and symmetric modes $\frac{\omega_a}{\omega_s}$.

Given $K = \frac{1}{LC}$ & $k^1 = \frac{1}{LC_1}$ (2013)

5. When connected in series L_1, C_1 have the same resonant frequency as L_2, C_2 also connected in series. Prove that if all these circuit elements are connected in series, then new circuit will have the same resonant frequency as either of the circuits first mentioned. (2014)

6. For initial assent conditions $I = I_0, \frac{dI}{dt} = 0$ at $t=0$, show that the time dependent current in the

critical damping case for an LCR circuit is $I = I_0 \left(1 + \frac{rt}{2}\right) e^{-rt/2}$

Where $r = \frac{R}{L}$, $\omega_0^2 = \frac{1}{LC}$ $\omega = \sqrt{\omega_0^2 - \frac{r^2}{4}}$ & $\tan \delta = \frac{-r}{2\omega}$ (2014)

7. A series RLC circuit has a resistance of 100 Ω and an impedance of 210 Ω . If this circuit is connected to an a.c. source with an r.m.s. Voltage of 220 V, how much is the average power dissipated in the circuit? (2015)
8. A series RLC circuit has $R = 2 \Omega$. The energy stored in the circuit decreases by 1% per period of oscillation. Its natural undamped frequency is 2 kHz. Determine the values of inductor L and the quality factor. (2015)

9. An alternative current varying sinusoidally with a frequency of 50 Hz has an r.m.s. value of 40 A. Find the instantaneous value of the current at 0.00125 second after passing through maximum positive value. (2016)

10. Describe the oscillations of electric and magnetic field in an ideal LC circuit. The applied voltage phasor in a circuit $(4 + 3i)$ volt and resulting current phasor is $(3 + 4i)$ ampere. Draw the phasor diagram. Determine the impedance of the circuit and indicate whether it is inductive or capacitive in nature. Also find the power dissipation in the circuit. (2020)

11. A $10\ \Omega$ resistor is connected in series with a capacitor of $1.0\ \mu\text{F}$ and a battery with emf 12.0 V. Before the switch is closed at time $t = 0$, the capacitor is uncharged.

Calculate the following :

(i) The time following :

(ii) What fraction of the final charge is on the plates at the time $t = 46$ seconds ?

(iii) What fraction of the initial current remains at the time $t = 46$ seconds ?

Consider that the internal resistance of the battery is zero and neglect the resistance of all the connecting wires. (2020)

TUTORIAL SHEET: 18**Electromagnetic Theory**

1. A wire of length 2 m is perpendicular to X-Y plane. It is moved with a velocity $\vec{V} = (2\vec{i} + 3\vec{j} + \vec{k})ms^{-1}$ through a region of uniform induction $\vec{B} = (\vec{i} + 2\vec{j})W m^{-2}$. Compute the potential difference between the ends of the wire. (2010)
2. Using Maxwell's field equations for a homogeneous non-conducting medium derive the wave equation for the electric field. Calculate the velocity of EM wave in free space. (2010)
3. Explain the term 'Poynting vector' and state the significance of Poynting theorem. (2010)
4. Calculate the skin depth for radio waves in free space of wavelength 3 m in copper, given that electrical conductivity for copper is $6 \times 10^7 \Omega^{-1}m^{-1}$. (2010)
5. The electric field of a plane e.m. wave travelling along the z-axis is $E = (E_{ox}\vec{i} + E_{oy}\vec{j}) \sin(\omega t - kz + \phi)$. Determine the magnetic field. (2013)
6. Considering an isotropic, linear, non-conducting, non-magnetic & inhomogeneous dielectric medium with $D = \epsilon E = \epsilon_0 n^2(x, y, z) E$. Show that EM wave equation for the field E is $\nabla^2 E + \nabla \left(\frac{1}{n^2} \nabla n^2 \cdot E \right) - \mu_0 \epsilon_0 n^2 \frac{\partial^2 E}{\partial t^2} = 0$.
(ii) Write down scalar equation for E_x from the above equation
(iii) Interpret physically the situation if we move from homogeneous to inhomogeneous medium (2013)
7. Using Amperes law and continuity equation, Show that the divergence of the total current density is zero. (2014)
8. Show that the energy flow due to a plane EM Wave propagating in Z direction in a dielectric media is given by $\frac{k}{Z\omega\mu} E_o^2 \cos^2(kz - \omega t)$, where k and ω are the propagation vector and angular frequency, E_o is electric field amplitude, μ is the relative permeability of the medium. (2014)

9. Starting from Maxwell's equation, Obtain the wave equation for the electric field E in free space and appropriate wave equation for the electric field $E = E_z(x, y, z)\hat{z}$. (2014)
10. Derive the equation that represents Poynting's theorem. What is its physical significance? (2015)
11. A radio station transmits electromagnetic waves isotropically with an average power of 200 kW . Determine the average magnitude of the maximum electric field at a distance of 5 km from it. (2015)
12. A plane electromagnetic wave propagating along $+\hat{z}$ direction is incident normally on the boundary at $z=0$ between medium $A(z<0)$ and medium $B(z>0)$. Determine the reflection coefficient and transmission coefficient for the wave. (2015)
13. What are the characteristic features of Rayleigh scattering? A very thin monochromatic beam of light is incident on a particle. Suggest a simple experimental method to ascertain whether the scattering by the particle is of Rayleigh type. (2015)
14. In free space, the electric field of electromagnetic wave is given as $\vec{E}(x, t) = 120 \cos(\omega t - kx)\hat{y}$ V/m. Find the average power crossing a circular area of radius one meter in the yz -plane. (2016)
15. Write down Maxwell's equations for linear dielectrics and deduce the equation of continuity. (2016)
16. State and prove Poynting's theorem. (2016)
17. Show that the displacement current between the plates of a parallel – plate capacitor is equal to the conduction current across the conductor. (2016)
18. A Parallel – plate capacitor is connected to a 240 V AC supply having angular frequency of 300 rad/s . Find the r.m.s. value of the conduction current in the circuit. What is the displacement current between the plates of the capacitor? Given, $C = 200\text{ pF}$. (2016)
19. Find the values of E and H on the surface of a wire carrying a current. By computing the Poynting vector, show that it represents a flow of energy into the wire. (2019)

- 20.** Briefly outline the theory of scattering of electromagnetic radiation by bound electron and hence derive the conditions for Rayleigh scattering. How can you explain the blue of the sky?
(2019)
- 21.** For the electric field given by $E = E_0 e^{i\omega t}$, show that the conduction current is in phase with the electric field, while the displacement current leads the electric field by $\frac{\pi}{2}$ radians. Also, show that the displacement current in a good conductor is negligible compared to the conduction current at any frequency at any frequency lower than the optical frequencies ($f < 10^{15}$ Hz).
(2020)
- 22.** For free space show that electromagnetic (EM) wave is transverse in nature. Show that for free space, the total outward flux of EM energy through surface S bounding volume V is equal to the rate of loss of EM energy from the volume V.
A laser beam of 2 mm diameter has average power of 20 GW. Calculate the peak values and magnetic fields in the laser beam.
(2020)
- 23.** Write maxwell's equations in free space in both differential and integral forms. Obtain Wave equations and show that electromagnetic wave can travel in free space with a speed of light, Can one get the wave equations from the integral form the Maxwell's equations ?
(2020)

TUTORIAL SHEET: 19**Black Body Radiation**

1. State and explain Stefan - Boltzmann's law show that $\log P = \log K + 4 \log R$, where P is the power emitted by blackbody and R is the resistance of the blackbody, K is Constant.
(2014)
2. In deriving radiation laws, we consider a cubical container of volume V containing a photon gas in equilibrium. Calculate the differential number of allowed normal modes of frequency W.
(2014)
3. Using Planck's radiation law, deduce Wien's displacement law. How does this law enable one to estimate the surface temperature of the Sun or a star?
(2015)
4. Two spheres A and B having same temperature A are kept in the surrounding of temperature T_0 . Consider $T > T_0$. The sphere are made of same material but have different radii r_A and r_B . Using Stefan – Boltzmann distribution, determine which of these will lose heat by radiation faster.
(2015)
5. The spectral energy curve of the moon shows maxima at 470 nm and 14 μ m. What interference can you draw from this data? Also calculate the energy density and radiation pressure in both cases. Given, Wien's constant $b = 2.892 \times 10^{-3} \text{ m K}$, Stefan's constant $\sigma = 5.67 \times 10^{-8} \text{ Jm}^{-2} \text{ s}^{-1} \text{ K}^{-4}$ and speed of light $c = 3 \times 10^8 \text{ m s}^{-1}$.
(2016)
6. Briefly explain Planck's law of blackbody radiation. Show that Planck's law reduces to Wien's law and Rayleigh – Jeans law at lower and higher wavelength limits respectively.
(2016)
7. Discuss in brief the ultraviolet catastrophe. How did Planck solve this problem? (2019)

PHYSICS PAPER - I: THERMAL PHYSICS**TUTORIAL SHEET: 20****Basic Concepts/Laws of Thermodynamics**

1. A thermally insulated ideal gas is compressed quasi statically from an initial state with volume V_o and pressure P_o to a final state volume V_f and pressure P_f , show that the work done on the gas in the process is given by $W = \frac{C_v}{R} (P_f V_f - P_o V_o)$ (2013)
2. State the first law of thermodynamics for a diffusively interacting system. The temperature of 10 g of air is raised by 2°C at constant volume. Calculate the increase in its internal energy.
Given : $C_v 0.172 \text{ cal g}^{-1} 2^\circ\text{C}^{-1}$ (2020)

TUTORIAL SHEET: 21

Entropy and Heat Engines

1. 1 kmol of an ideal gas is compressed isothermally at 400 K from 100 k Pa to 1000 k Pa in a piston and cylinder arrangement. Calculate the entropy change of the gas, of the surroundings and the total entropy change resulting from the process if the process is mechanically reversible and the surroundings consist of a heat reservoir at 400K. **(2010)**
2. Derive an expression for the thermal efficiency of a reversible heat engine operating on the Diesel cycle with an ideal gas of constant heat capacity as the working medium. **(2010)**
3. The vapor pressure in mm of Hg of a substance in solid state is given by the relation in $P = 23 \cdot 03 - \frac{3754}{T}$. The vapor pressure in mm of Hg of the substance in liquid state is given by $\ln P = 19 \cdot 49 - \frac{3063}{T}$. Calculate
 - ii. The coordinates of the triple point
 - iii. The latent heat of vaporization at the triple point **(2013)**
4. In Leh, the temperature of ice on cold winter night is measured as -20°C . Calculate the change in entropy when 1kg of ice is converted into steam at 100°C . Given specific heat capacity of ice is 500 cal/kg, latent heat of ice is $3.36 \times 10^5 \text{ J/kg}$, latent heat of steam is $2.26 \times 10^6 \text{ J/kg}$. **(2013)**
5. One kg of water at 20°C is converted into ice at -10°C at constant pressure. Heat capacity of water is 4200 J/kg/K and that of Ice is 2100 J/kg/K. Heat of fusion of Ice at 0°C is 335 x 10³ J/kg calculate the total change in entropy of the system. **(2014)**
6. m gram of water at temperature T_1 is isobarically and adiabatically mixed with an equal mass of water at temperature T_2 . Show that the change in entropy is given by
$$\Delta S = 2m C_p \ln \left(\frac{T_{av}}{T_{geo}} \right) \text{ where } T_{av} = \frac{T_1 + T_2}{2} \text{ and } T_{geo} = \sqrt{T_1 T_2}$$
(2016)
7. What is Carnot's theorem? Prove that Carnot's reversible engine is the most efficient one and no other engine can be more efficient than Carnot's engine. **(2019)**

TUTORIAL SHEET : 22**Thermodynamic relationships**

1. Calculate the change in pressure for a change in freezing point of water equal to -0.91°C . Given, the increase of specific volume when 1 gm of water freezes into ice is 0.091 cc/gm and latent heat of fusion of ice is 80 cal/gm . (2010)
2. Establish the relation $\left(\frac{\partial T}{\partial V}\right)_P = -\left(\frac{\partial P}{\partial S}\right)_T$ and then derive $\left(\frac{\partial C_P}{\partial P}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_P$. Hence show that the heat capacity C_P of an ideal gas is independent of pressure P. (2012)
3. Show that the Helmholtz free energy of a system never increase in any isothermal isochoric transformation. (2012)
4. Define Enthalpy and show that it remains constant in throttling process. (2014)
5. The vapor pressure of an organic substance is $50\times 10^3\text{ Pa}$ at 40°C . Its normal boiling point is 80°C . If the substance in vapor phase can be treated like an ideal gas, Find the latent heat of vaporization of the substance. (2015)
6. A Van der Waals gas undergoes Joule-Kelvin expansion with a pressure drop of 50 atm . If its initial temperature is 300°K , determine its final temperature. (Given Van der Waals constant $a = 0.136\text{ Pa m}^6\text{ mol}^{-1}$, $b = 36.5\times 10^{-6}\text{ m}^3\text{ mol}^{-1}$, $C_p = 30\text{ JK}^{-1}\text{ mol}^{-1}$, $R = 8.3\text{ JK}^{-1}\text{ mol}^{-1}$) (2015)
7. What do you understand by the term from phase transition? Using Clausis-Clapeyron equation, Show that for first-order phase transitions, vapour pressure decrease exponentially with temperature. You can assume that the vapour behaves like an ideal gas and latent heat remains constant with temperature. (2016)
8. What are the conditions for the change in temperature of a van der Waals gas passing through a porous plug? Prove that the ideal gas passing through the porous plug does not show any change in temperature. (2019)
9. Explain the effect of pressure on the melting and boiling points of a substance using Clapeyron's latent heat equation.

Calculate under what pressure, water will boil at 120°C , if the change in specific volume when 1 gram of water is converted into steam is 1676 cm^3 . Latent heat of steam = 540 cal/g , 1 atmospheric pressure = 10^6 dynes/cm^2 . **(2019)**

- 10.** Obtain the Clausius-Clapeyron equation. Using this equation, show that for the phase boundary of the liquid and vapour phases, p-T relation can be written as $p = p_0 e^{-L/Kt}$. Here it has been assumed that the latent heat L is temperature, that vapour is treated as an ideal gas and that $V_{\text{vapour}} = V \gg V_{\text{liquid}}$ and that $P \rightarrow P_0$ as $T \rightarrow \infty$. **(2020)**

TUTORIAL SHEET: 23**Adiabatic Demagnetisation/ Special topics**

1. The coefficient of viscosity of helium at 27°C is $2 \times 10^{-5}\text{kg/m/sec}$. Calculate
 - i. Average speed
 - ii. The diameter of a helium molecule, If it is assumed that the gas obeys Maxwell – Boltzmann distribution man of the helium atom = $6.67 \times 10^{-27}\text{ kg}$. **(2013)**
2. For a Vander Waal's gas, write down the equation of state. Determine the coefficient of critical expansion. **(2015)**
3. Write down van der Waal's equation of state for n moles of a gas and calculate the temperature at which 5 moles of the gas at 5 atm pressure will occupy a volume of 20 litres. Given $R = 8.31 \times 10^7 \text{ erg mol}^{-1}\text{K}^{-1}$, $a = 1.34 \times 10^{12} \text{ dyne cm}^4\text{mol}^{-2}$, $b = 31.2 \text{ cm}^3 \text{ mol}^{-1}$ and $1 \text{ atm} = 1.013 \times 10^6 \text{ dyne cm}^{-2}$. **(2016)**
4. What is Gibbs' phase rule? Find the values of degree of freedom when
 - (i) Only the liquid CO_2 is in equilibrium with the gaseous CO_2
 - (ii) Water is in the vapour-liquid saturation region.
 - (iii) Water is in a single-phase region,
 - (iv) Water is at the triple point. **(2019)**
5. Discuss the principal adiabatic demagnetization process to achieve low temperatures. Determine the fall in temperature produced by adiabatic demagnetization of a paramagnetic material at initial temperature of 3 K when the magnetic field is switched off from 10,000 oersted to zero. Given : heat capacity at constant magnetic field = $0.2 \text{ J g}^{-1} \text{ K}^{-1}$ and Curie constant per gram mole per $\text{cm}^3 = 0.042 \text{ erg K}^{-1} \text{ g}^{-1} \text{ Oe}^{-2}$. **(2020)**

TUTORIAL SHEET: 24

Specific heat of solids

1. The Einstein theory of specific heat of solids given the expression $C_V = 3NK_B \frac{e^x x^2}{(e^x - 1)^2}$

Where $x = \frac{\theta_E}{T}$

- i. Mention Einstein's assumptions in deriving it and obtain low and high temperature limit expressions for it.

- ii. Give schematic plot of $\frac{C_V}{3NK_B}$ Versus $\frac{T}{\theta_E}$ and comment on the validity of expression in comparison with experiments (2012)

2. Einstein's molar specific heat capacity of a solid is given by

$$C_V = 3R \left(\frac{\theta_E}{T} \right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T} - 1)^2}$$

Where $\theta_E = \frac{\hbar\omega}{k_B}$

Obtain the expressions for the cases:

- (i) When $T \gg \theta_E$
(ii) When $T \ll \theta_E$

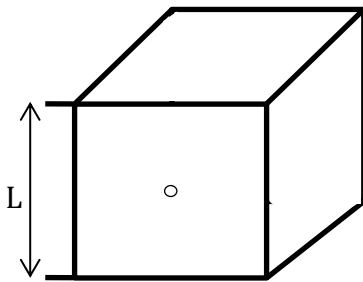
What is the discrepancy of Einstein model to explain the variation of specific heat capacities of solids with the temperature?

The molar specific heat capacity of a solid at constant volume is 2.77 JK^{-1} at 36.8 K.

Determine the Debye temperature of the solid. (2019)

3. A historic failure of Classical Physics is its inability to describe the electromagnetic radiation emitted from a black body. Consider a simple model for an ideal black body consisting of a cubic cavity of side L with a small hole on one side.

- (i) Assuming the classical equipartition of energy, derive an expression for the average energy per unit volume and unit frequency range. In what way does this result deviate from actual observation? What is this law called?



(ii) Repeat the Calculations now using quantum idea to obtain an expression that properly accounts for the observed spectral distribution .

Find the temperature dependence of the total power emitted from the hole. **(2020)**

TUTORIAL SHEET : 25

Maxwell- Boltzmann's Statistics

1. N particle obeying classical statistics are distributed among three energy states $\epsilon_1 = 0, \epsilon_2 = KT$ and $\epsilon_3 = 2KT$. If the total equilibrium energy of the system is $1000 KT$, calculate the value of N. (2013)
2. Using Maxwell Boltzmann distribution law prove that there cannot be any negative absolute temperature. (2014)
3. Consider a system of free gas particle having f degree of freedom. Use equipartition theorem
$$f = \frac{2}{\frac{C_p}{C_v} - 1}$$
to establish the relation
Where C_p and C_v are molar specific heats at constant pressure and constant volume respectively. Obtain the values of $\frac{C_p}{C_v}$ for diatomic and triatomic gases. (2014)
4. The molecules of a gas obey Maxwell-Boltzmann distribution. Calculate the fraction of molecules of the gas within 1% of the most probable speed at STP. Interpret your result? (2016)
5. Consider a system of N particles and a phase space consisting of only two states with energies 0 and ϵ (> 0). Obtain the expressions for the partition function and the internal energy of the system. If it obeys M-B statistics. (2016)
6. The molecules of a gas obeying Maxwell-Boltzmann distribution, move with an average speed of 450 ms^{-1} . If the coefficient of viscosity of the gas η is $16.6 \times 10^{-6} \text{ N s m}^{-2}$, density of the gas ρ is 1.25 kg m^{-3} and number density is $2.7 \times 10^{25} \text{ m}^{-3}$. Calculate the mean free path and diameter of the gas molecules. (2016)
7. The viscosity in a liquid arises due to friction between adjacent layers. What causes viscosity in a gas? Explain. (2016)
8. If the partition for a perfect gas is given by
$$Z = \frac{V}{h^3} (2\pi mkT)^{3/2}$$
Calculate (i) average kinetic energy per molecule and (ii) specific heat of the gas. (2019)
9. The energy level of a quantum harmonic oscillator with frequency ν is given by

$$E_n = \left(n + \frac{1}{2}\right) h\nu, \text{ where } n = 0, 1, 2, \dots$$

Calculate its partition function .

(ii) Calculate the partition function of a two level system.

(2020)

10. Starting from Maxwell-Boltzmann distribution for a free particle in 3-dimension, obtain the expression for root mean square (rms) speed of a particle. Calculate the rms speed of nitrogen (N_2) molecule at room temperature ($27^\circ C$).

(2020)

TUTORIAL SHEET: 26

F-D & B-E Statistics

1. Consider the following statement:

“The Fermi energy of a given material is the energy of that quantum state which has the probability equal to $\frac{1}{2}$ of being occupied by the conduction electrons.”

Is the above statement correct? Give reasons for your answer. (2010)

2. Show that both FD and BE distributions reduce under certain conditions in a form which

gives the total number of particles as
$$N = A \int_0^\infty \sqrt{\epsilon} e^{-\beta \epsilon} d\epsilon$$
, where A is a constant and $\beta = \frac{1}{K_B T}$ Further show that this expression is just the same as obtained from the Maxwellian speed distribution (2012)

3. Show that both Fermi – Dirac and Bose – Einstein distribution functions at an energy E are

given by $f(E) \simeq \exp \frac{(\mu - E)}{K_B T}$, Where $f(E)$ is much smaller than unity, μ and $K_B T$ are the chemical potential and thermal energy of the atom. (2014)

4. A gas has only two particles, a and b. With the help of a diagram, show that how these two particles can be arranged in the three quantum series 1, 2, 3 using (i) Maxwell-Boltzmann, (ii) Fermi-Dirac, and (iii) Bose-Einstein statistics. (2019)



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